Targeted Information and Limited Attention

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June 2018

Abstract

We examine the implications of limited consumer attention for the targeting decisions of competing firms. Limited attention alters the strategic role of information provision as firms may be incentivized to behave as mass advertisers, despite potentially perfect targeting abilities. We analyze the consequences of limited attention for market shares, information taxation, attention competition between firms, the value of marketing data to firms and strategic pricing. Accounting for limited attention in an otherwise standard targeting framework can explain several recent key issues from the advertising industry, such as increased consumer ad blocking, privacy concerns and information overload.

Keywords: targeted advertising, limited attention, ad avoidance, salience competition, privacy concerns

JEL classification: D43; L13; M37

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1 Introduction

Modern information systems, above all the Internet, allow sellers to gather an enormous amount of data about their potential consumers. The availability of such data allows firms to target their ads towards precisely identified subsamples of consumers. Advertising research leaves little doubt that targeted information has become a dominant source of advertising revenues. Large businesses such as Axciom, IRI or Nielsen make their revenues by selling consumer data to individual companies. Sponsored search advertising, allowing firms to advertise towards consumers who indicate an interest by their web search queries, has become "the largest source of revenues for search engines" (Ghose and Yang, 2009). According to the annual report by the Interactive Advertising Bureau (IAB), search advertising alone already has a steady 40% of total digital ad revenues, which was about $6.76 billion in 2006 and nearly trippled to $18.81 billion in 2014. Similarly, sponsored advertising has grown annually by around 30% from $1.12 billion in 2011 to $1.88 billion in 2013, and is expected to grow further.[1]

Given the unprecedented array of customer information and information sharing technologies, including email, SMS, tweeting and social networks, it may come as a surprise that several recent press releases shed doubt on the extent to which real-world targeting behavior really benefits both firms and consumers.[2] If the data about consumers is so comprehensive and detailed, and tailored advertising opportunities are as easy and cheap as never before,[3] why do many consumers complain about being overloaded with ads that are not most relevant to them, while firms appear to target their messages far less precisely than they could?[4] This contrasts with the conjecture, voiced a decade ago by a principal analyst of Forrester Research, that the possibility to send targeted messages should lead to "the end of the era of mass marketing" because "nobody can afford it anymore, and consumers are overloaded with messages".[5] In-

[1]Data available online on www.iab.com/insights and www.emarketer.com. See Yao and Mela (2011) for similar facts on the importance of sponsored search in advertising, and Evans (2009) for a general survey on advertising data.

[2]Examples include the articles "Does sponsored content work for advertisers?", *Wall Street Journal*, March 23, 2014 and "Does targeting work?", *The Ad Contrarian*, Feb 01, 2012, and the IAB report "Online consumers view and usage of ad blocking technologies", Sep 2014. Farahat (2013) shows that previous studies on targeted advertising may have largely overestimated its effectiveness due to not accounting for selection bias.

[3]See, e.g., "How online advertisers read your mind", *The Economist*, Sep 2014.

[4]See, e.g., "Adespresso.com" for a practitioner's report.

[5]"Cruise ship campaign aims at vacationers tired of snow", *The New York Times*, Dec 28, 2004.

deed, the observation of information-loaded markets with overwhelmed consumers is hard to reconcile with the theoretical prediction of how competition should shape the firms' targeting decisions. It is commonly understood that competition forces the advertising firms to concentrate their messages on those consumers to which each firm offers the best match (Iyer et al., 2005; Esteban and Hernandez, 2007; Brahim et al., 2011; Esteves and Resende, 2016). With perfect targeting abilities, firm targeting may coincide with the distribution of preferences, which yields an efficient information dissemination. Moreover, consumers can be faithful that the market will identify their best options for them. This makes targeted advertising potentially a highly effective marketing instrument, especially in view of the inefficiencies associated with mass advertising (Grossman and Shapiro, 1984), but is at odds with how real-world targeting has evolved.

In this article, we relax the standard (implicit) assumption that consumers are endowed with unlimited attention capacities. Considering limited attention seems both natural and important given the obvious superabundance of information in the modern digitized economy, mirrored by the dominant preoccupation in the advertising industry that a seller's messages could simply be overlooked by consumers.[6] Our main finding is that several real-world phenomena related to targeting can be explained within an otherwise standard targeting framework once limited consumer attention, i.e., an upper bound on the perceptible alternatives, is taken into account.

Baseline model To understand the effects of limited consumer attention in a targeting context, we first study a simple, stylized horizontal duopoly model with ex ante uninformed consumers. Firms, being endowed with complete information about consumer preferences and perfect targeting abilities, need to choose their costly targeting strategies, i.e., the subset of consumers to receive information. Consumers always choose the best product they perceive. Attentive consumers always see all ads targeted at them, while firms have exogenously given perception chances for mutually targeted, inattentive consumers, who perceive just one firm in the end. This simple model is sufficiently tractable to yield a complete characterization of how equilibrium targeting depends on the measure of inattentive consumers, consumer tastes, product characteristics and firm-side information costs. Section 2 establishes that consumer

[6]See, e.g., "Advertising and technology", *The Economist*, Sep 2014, Special Report.

inattention can have a severe impact on the targeting behavior. In particular, firms may strategically behave as if they were mass advertisers, sending their messages to every consumer, despite the availability of perfect data about consumer tastes and an infinitely precise targeting technology. This is antipodal to the prediction resulting from the standard assumption that consumers have unlimited attention capacities, where competition disciplines both firms to send their messages exclusively to their prime segments, i.e., to those consumers who find the respective firm to be their best choice. The main intuition is that, besides informing the consumer about the product, information targeted at prime consumers works as an efficient shield against wasteful business-stealing, provided that consumers are able to register and memorize any piece of information they see. This simple mechanism deters each firm from intruding the competitor's prime segment, even with possibly heterogeneous and arbitrarily small information costs. The presence of inattentive consumers may dispel this protective role of information provision, because firms have an incentive to invade the competitor's prime segment in an attempt to absorb consumer attention. More generally, we identify four possible and mutually exclusive types of targeting equilibria, together with their characteristic parameter constellations. The standard, efficient *segmenting equilibrium* arises either if inattention is negligible or information costs are moderate, but never if almost every consumer is inattentive. With enough inattentive consumers, one of three possible inattention-specific equilibria occurs, depending on the firms' (relative) abilities to attract attention, and information costs. If both firms face low information costs relative to their attention-seeking abilities, the *mass advertising equilibrium* results, while high information costs imply that firms encounter a coordination problem with a continuum of possible, and generically inefficient, *coordination equilibria.* With sufficiently asymmetric information costs a *partial mass advertising equilibrium* is predicted, where the low cost firm mass advertises and, depending on the parameters, the high cost firm either targets only her prime segment or shuts down.

The baseline model identifies further consequences of limited attention for market shares and welfare. If the degree of inattention increases, traditional fundamentals, such as preferences or product differentiation, become less important for determining the equilibrium market shares. From the normative viewpoint, inattention leads to economic inefficiency, either because of excessive targeting in the mass advertising equilibrium, or because of generic mis-targeting in case of the coordination equilibria. Consequently, a small mass of inattentive consumers may impose a substantial, discontinuous negative externality on attentive ones if the equilibrium switches from segmenting to mass advertising, which is related to what Armstrong (2015) calls a search externality.

We further investigate whether an information tax (or subsidy) levied on attention-seekers could resolve the informational inefficiency associated with the inattention-specific equilibria. Such a tax scheme for online advertisers was repeatedly called for by the French government in recent years.[7] We find that the scope for such a tax can be very small (or even inexistent) with many inattentive consumers or very asymmetric firms. We also show that our results are robust to pay-per-click costs, where coordination equilibria become less and the mass advertising equilibrium more likely to occur with such a cost structure.

Extensions We relax several assumptions of the baseline model. These variations provide additional insights, but the essential way how limited attention affects equilibrium targeting remains the same as before.

In section 2.2.1 we introduce *strategic salience competition* to endogenize the chance that a firm is perceived by inattentive consumers. Firms can costly manipulate the likelihood of grabbing the attention from mutually targeted consumers, which possibly interferes with the underlying targeting decision. We establish that the resulting competition for attention endogenizes the information costs in such a way that the coordination equilibria cannot emerge, making mass advertising the unique equilibrium prediction. As we further show in Appendix A.5, mass advertising remains the unique equilibrium prediction even if we allow for taste-depending salience effect.

Section 2.2.2 introduces price competition to the targeting framework. In view of the extremely transient, flexible nature of online advertising in the digital economy, most pertinent in case of real-time ad space auctions, we study the two-stage variant where firms first decide on their pricing policy and then choose their targeting strategies. As in the baseline model, limited attention implies the mass advertising (coordination) equilibrium with low (high) infor-

[7]See, e.g., the article "France proposes an Internet tax", *The New York Times*, Jan 20, 2013.

mation costs, while the standard information segmentation equilibrium results with attentive consumers. The fact that equilibrium prices are higher compared to the full-attention benchmark reveals an additional consumer welfare loss in the mass advertising equilibrium. Likewise, a *higher* information cost (e.g., caused by a tax) can be beneficial to firms with inattentive consumers because it makes them commit not to penetrate their opponent's market. Moreover, we find a different impact on how certain consumer characteristics influence equilibrium pricing and profits in equilibria with limited consumer attention.

In section 2.2.3 we relax the assumption that firms hold ex ante perfect marketing data. We obtain the same insight how limited attention matters for targeting as in the baseline model, albeit with an additional qualification. A limited quality of the marketing data implies some overlap of the targeting profiles also in case of attentive consumers. Nevertheless, this overlap only reflects limitations in the data and not a strategic desire to intrude the competitor's prime segment, which is the driving force of intrusive targeting with limited attention. Thus, the generalized equilibrium targeting behavior with attentive consumers can best be described as that firms seek to exclusively target their prime segments *as indicated by their available data*. This implies that the incentives for information acquisition depend on consumer attention. With attentive consumers, firms always benefit from more precise marketing data, allowing them to target more efficiently. Consequently, better marketing data yields less overlap in the targeting profiles, and rational consumers were willing to reveal their tastes, anticipating that the market uses this information in their best interest. In contrast, limited attention makes such marketing data far less important once firms resort to mass advertising, and consumers cannot benefit from sharing their information with the firms.

Ad blocking In section 3 we study the consequences of limited attention when consumers have the option of using ad blockers. The recent remarkable increase in the usage of ad blockers has become a serious challenge to the entire advertising industry. According to the report "The cost of ad blocking" by PageFair and Adobe (2015) the year 2014 has seen a 48% increase of ad blocking in the US. The estimated revenue loss to the industry is $21 billion, or 14% of global ad spending. The IAB also reported that more than 1/3 of US adults use ad blockers.[8]

[8]See "Online consumers views and usage of ad blocking technologies", IAB report, Sep 2014.

Both studies identify an increase in the exposure to ads together with an intrusive view on advertising to be among the main reasons why people start using ad blocking software.[9] This increased blocking came as a surprise to the ad industry, especially since the involved blocking technology has ostensibly not improved much during the last decade.[10] While the blocking development is indeed unexpected from the viewpoint of standard theory – we show that if consumers have unbounded attention capacities such blocking should not arise with strong targeting abilities – it is the main prediction with attention-constrained consumers. The main reason is as follows. Consumers face a trade-off between accepting a certain nuisance caused by incoming ads and forgiving consumption utility by blocking information. Rational consumers choose to block iff the anticipated nuisance of advertising exceeds the expected benefits of the perceived information. Accordingly, a consumer's ad blocking decision depends on the anticipated information structure of the targeting equilibrium. As in the baseline model, the firms are disciplined to target their messages exclusively at prime consumers in case of unlimited attention. In response, consumers, anticipating that i) they will only receive a limited amount of information and ii) the retrieved information is most relevant to them, thus have little incentives to use ad blockers. However, if consumers came to believe that the market fails to make them perceive information with enough value to compensate for the nuisance of the incoming advertising, they would become inclined to block. For this reason, nuisance-sensitive consumers choose to block once they expect the mass advertising equilibrium to occur, while the firms remain locked in their role as mass advertisers, despite that their behavior leads to consumer blocking and, hence, forgone sales.

Section 3.2, pooling together the main insights, illustrates that our results can help explain the coexistence of (increased) ad blocking and consumers voicing concerns about information overload and privacy. The sentiment of information overload results from mass advertising jointly with the nuisance from endogenous salience competition, leading to the ad blocking of some consumers. Further, privacy concerns are likely to be expressed once consumers fear that

[9]These findings are confirmed by the IAB report "B2B ad blocker study of the OVK", Oct 2015, in case of Germany. This study lists retargeting among the most important reasons for the use of ad blockers.he Also see "Invisible ads, phantom readers", *The Economist*, Mar 2016.

[10]See online articles "Does targeted advertising work in 2015" by Jay Dillon, the director of the marketing agency Inboundly, and "ClarityRay battles ad blockers with $500K in funding", May 2012. For company advice how to respond to increased ad blocking, see "Ad blocking: the unnecessary Internet apocalypse", Sep 2015.

information about them is not used to their best interest by the market. The incentive of consumers to share their data with firms entirely breaks down in the mass advertising equilibrium caused by limited attention, where targeting is merely driven by attentional concerns. If consumers face an arbitrarily small cost of revealing their type, they would always be reluctant to share their information in such a situation, making privacy concerns arise naturally. We thus identify an additional channel of inefficiency due to limited attention, which closely relates to the worries of many ad consultancies and is particularly troublesome given that there is social surplus from any match between firms and consumers in our model.[11]

Appendices Appendix A contains several additional comments on the model, and further studies the cases of taste-sensitive salience effects and persuasive advertising. Appendix B contains the proofs of formal theorems from the main text.

1.1 Related literature

Our paper primarily contributes to the literature on targeted advertising. To our knowledge, this is the first paper that seeks to study limited consumer attention within an otherwise standard competitive targeting framework. Esteban et al. (2001) study how the technology of targeted advertising affects the outcomes in a monopoly market. In case of homogeneous products, Roy (2000) shows how firms can soften price competition by providing targeted information to consumers. Several papers (e.g. Iyer et al., 2005; Esteban and Hernandez, 2007; Esteves and Resende, 2016) study targeted advertising with differentiated products. All these papers (implicitly) assume that consumers are fully attentive, which implies that a firm would never want to target a consumer who is aware of better-matched products. Our paper highlights that this core feature of firms' targeting incentives does not apply with attention-constrained consumers. This implies that, in contrast to the broad prediction of the above papers, the inefficient mass advertising equilibrium outcome ought to prevail even if firms have perfect targeting abilities.

[11] Previous studies on the determinants of the consumers' willingness to share information with advertisers have mainly focused on the sensitivity of the information and the scope of the collection and use of information (see, e.g., Nam et al., 2006; Ur et al., 2012; Leon et al., 2015).

More related to our paper are Van Zandt (2004) and Brahim et al. (2011), albeit for different reasons. Van Zandt (2004) studies a model of targeted information with non-competitive products, where consumers are only able to sample a subset of targeted products with exogenous and uniform probabilities. In contrast, we study the case of competitive products with the possibility of non-uniform perception chances, and introduce both price and salience competition to the model. Moreover, the limited attention prediction is contrasted to its standard counterpart. Brahim et al. (2011) consider a Hotelling duopoly with attention-unconstrained consumers, where each firm independently chooses an advertising intensity for each targeted consumer i, which is the probability that i receives the ads. They find that firms target all consumers, but never at full intensity, if targeting costs are high enough, while each firm confines its targeting to its prime segment with low costs. This result is driven by the implicit assumption of fully attentive consumers. Firms can benefit from targeting non-prime consumers only if these consumers may miss the better option. High targeting costs generate such a situation, because it simply is too expensive to advertise at full intensity to any consumer. With low targeting cost, each firm targets only its prime segment at maximal intensity, which then works as a protective shield against business stealing exactly because consumers have unlimited attention. In contrast, limited attention yields the exact opposite prediction in our framework: mass advertising is the unique equilibrium if targeting costs are *low*, while segmentation occurs for *high* targeting costs. Moreover, when targeting costs are endogenized by the firms' competition for consumer attention, mass advertising becomes the unique equilibrium prediction, and all consumers will be targeted by both firms at the *same* intensity.[12]

We also contribute to the literature on ad blocking (e.g., Armstrong et al., 2009; Anderson and Gans, 2011; Gritckevich et al., 2018), by studying a variant of ad blocking closely related to Johnson (2013).[13] In case of unlimited attention, we confirm Johnson (2013)'s prediction that a more precise targeting technology benefits the firms, while limited attention may trigger mass advertising jointly with an increased ad blocking by consumers in situations where, by

[12]This exemplifies the fundamental difference between "advertising intensity" and "salience competition", where the latter originates from the contest for attention as a scarce resource. In Brahim et al. (2011), the success of a firm's advertising intensity is invariant to the competitor's efforts, while with salience competition this is never the case.

[13]As Johnson (2013), we abstract from price competition in our analysis, noting that endogenous pricing as introduced in section 2.2.2 would not change results.

conventional channels, such blocking should not arise.[14]

More broadly, our paper contributes to the growing literature on behavioral industrial organization. Typically, papers in this literature are interested in how market outcomes are determined by the interaction between boundedly rational consumers and fully rational, profit-maximizing firms. Ellison (2006), Spiegler (2011), and Heidhues and Köszegi (2018) provide comprehensive surveys on related topics.

2 A Model of Targeted Information

In this section, we develop and analyze a baseline model of targeted information and limited attention (LA). We work with a Hotelling framework mainly because i) of its simplicity and familiarity, and ii) it allows to quantify welfare effects. The locational structure is not restrictive for our core results, however, as these apply to any model featuring some measurable assignment of preferential prime segments to consumers (see the discussion in Appendix A.1).

Consider two firms, indexed by $j \in \{A, B\}$, located on a Hotelling line $[0, 1]$. Let $x_j \in [0, 1]$ be the location of firm j, where $x_A < x_B$. A unit mass of consumers is uniformly distributed over the line. If a consumer $i \in [0, 1]$ transacts with firm j, she receives a benefit $U_i(j) = \mathcal{V}_j - t|i - x_j|$, where $\mathcal{V}_j$ is a consumer-independent value of j-consumption to i, while $t|i - x_j|$, $t > 0$, quantifies consumer-side (transportation) costs. Specifically, we let $\mathcal{V}_j = V_j - p_j$, where V_j reflects j's product quality and p_j is the price charged by j. To establish the consequences of limited attention for the targeting behavior as clearly as possible, we begin by abstracting from price competition.[15]

Suppose that $\mathcal{V}_j \geq t$, hence $U_i(j) \geq 0 \ \forall i \in [0, 1], j \in \{A, B\}$, and let $\mathcal{P}_j \equiv \{i \in [0, 1] : U_i(j) \geq U_i(-j)\}$, $j \in \{A, B\}$ denote firm j's *prime segment* of consumers. Then, $\mathcal{P}_A \cup \mathcal{P}_B =$

[14] In a different setup, Armstrong et al. (2009) also show that enabling information blocking can soften price competition between firms and negatively impact consumer welfare.

[15] One may note that several contributions on targeted advertising have neglected price competition (e.g., Van Zandt, 2004; Athey and Gans, 2010; Johnson, 2013). We encompass endogenous pricing in section 2.2.2.

$[0,1]$, and $\mathcal{P}_A \cap \mathcal{P}_B = \{i_0\}$, $i_0 \in (0,1)$, if and only if

$$\frac{|\mathcal{V}_A - \mathcal{V}_B|}{x_B - x_A} < t. \tag{1}$$

If (1) is satisfied, the segmentation point i_0 is

$$i_0 = \frac{\mathcal{V}_A - \mathcal{V}_B}{2t} + \frac{x_A + x_B}{2}, \tag{2}$$

and $\mathcal{P}_A = [0, i_0]$, $\mathcal{P}_B = [i_0, 1]$, accordingly. If (1) is violated, the prime segment of one firm coincides with the entire consumer population. Note that prime segments are entirely determined by traditional fundamentals, such as product characteristics and consumer preferences.

Information, limited attention, and consumer choice Let $X_i \subseteq \{A, B\}$ be consumer i's information set, which records from what firms a consumer receives information. Akin to the targeting literature, we assume that $j \in X_i$ iff consumer i has been informed by j.[16] The novel feature is that each consumer is described by her *attentiveness* $R_i \in \{1, 2\}$, where the capacity limit R_i is the maximal number of alternatives that i perceives.[17] We do not have in mind that $R_i = 1$ necessarily means that a consumer perceives only a single product, but rather only one menu of products, like an IKEA catalog, in which case j is the underlying brand.[18]

LA implies that received and *perceived* information may disagree. Let $A_i \subseteq X_i$ denote i's attention set, i.e., the subset of alternatives that is perceived at the time of purchase. Then, A_i and X_i are identical iff $X_i \subsetneq \{A, B\}$ or $X_i = \{A, B\}$ and $R_i = 2$. For consumers with limited

[16] Our results do not hinge on the assumption of ex ante uninformed consumers (see discussion in Appendix A.2).

[17] LA as a capacity threshold on the number of distinguishable objects has been used, e.g., by Van Zandt (2004), Anderson and De Palma (2009, 2012) and Hefti (2015). Other approaches to LA work with a volume-based threshold (Falkinger, 2007) or a threshold on the processable amount of bits in noisy signals (Sims, 2003). See Hefti and Heinke (2015) for a survey.

[18] Nevertheless, Gu et al. (2012) study the degree to which consumers consider several alternatives at the time of purchase in sales data of Amazon's electronic category, and retrieve that more than 78% of all consumers consider only a *single* product.

attention ($R_i = 1$), perception is mutually exclusive, where firm j's perception chance is

$$P(j \in A_i) = \begin{cases} 1 & \text{if } j \in X_i \text{ and } |X_i| = 1, \\ \pi_j & \text{if } j \in X_i \text{ and } |X_i| = 2, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

$|X_i|$ is the size of the information set, $\pi_A, \pi_B \in (0,1)$ and $\pi_A + \pi_B = 1$. A consumer always transacts with her best perceived firm if $A_i \neq \emptyset$, and not at all otherwise. We assume, for the moment, that the perception probabilities (or salience parameters) π_A, π_B are exogenous.

Firm strategies and costs Each firm needs to strategically choose which consumers to target. Formally, a targeting strategy is a (Lebesgue-)measurable function $g_j : [0,1] \to \{0,1\}$, where $g_j(i) = 1$ indicates that consumer i has been targeted by firm j, and accordingly $j \in X_i$. We denote the set of all measurable indicator functions on $[0,1]$ by $\mathcal{L}$, and the measure of a (measurable) set $S \subseteq [0,1]$ by $\lambda(S)$. Firm j's total expenditure on its information campaign is $c_j \lambda(\mathcal{I}_j)$, where $\mathcal{I}_j = \{i \in [0,1] : g_j(i) = 1\}$ is the set of consumers targeted by firm j, and $c_j > 0$ is the marginal information cost. This is a standard type of cost function in the targeting literature,[19] and we will discuss later how our main result can be adapted when an alternative pay-per-click cost structure (Hu et al., 2015) is considered instead. Finally, to make the core effects of LA on firms' targeting decisions most evident, we assume that i) each firm j is endowed with perfect information about each consumer's location and ii) earns an exogenous revenue $p_j > c_j$ for each successful transaction. Both assumptions are relaxed later in the paper.

2.1 Targeting Equilibrium

Firms simultaneously and non-cooperatively choose their targeting strategies to maximize expected payoffs. A *targeting equilibrium* is a targeting profile $(g_A, g_B) \in \mathcal{L}^2$, such that neither firm can gain a strict advantage by unilaterally deviating to any alternative targeting strategy.

[19]See, e.g., Van Zandt (2004), Iyer et al. (2005), Bergemann and Bonatti (2011), Johnson (2013) and others.

The following two distinguishable properties of a targeting profile are important in the later analysis.

Definition 1 *A targeting profile is intrusive if* $\exists j \in \{A, B\}$ *such that* $\lambda(\mathcal{I}_j \cap \mathcal{P}_{-j}) > 0$. *A targeting profile is overlapping if* $\lambda(\mathcal{I}_A \cap \mathcal{I}_B) > 0$.

Intuitively, a targeting profile is intrusive if a firm targets non-prime consumers, and it is overlapping if it features jointly targeted consumers. We say that an equilibrium is non-intrusive (non-overlapping) if its targeting profile is not intrusive (not overlapping). Overlapping targeting is sufficient but not necessary for intrusive targeting in our framework.

We suppose that attention capacities R_i are distributed over the consumer population according to iid draws with $\Pr(R_i = 1) = q$ and $\Pr(R_i = 2) = 1 - q$, where $q \in [0, 1]$. The parameter q thus is the measure of inattentive consumers in the market. The main theorem of this section characterizes the four equilibrium types that arise in the targeting game, depending on the degree of inattentiveness (q), the firms' abilities to attract attention-constrained clients (π_A, π_B), and information costs.

Theorem 1 (Targeting equilibria) *Let* $p_j > c_j > 0$ *and* $\pi_j \in (0, 1)$ *for* $j = A, B$.

(i) ***Segmenting*** *If* $c_j > q\pi_j p_j$ *and* $c_j < q\pi_j p_j + (1-q)p_j$ $\forall j \in \{A, B\}$, *then any targeting equilibrium is non-intrusive, i.e.,* $\lambda(\mathcal{I}_j \cap \mathcal{P}_{-j}) = 0$, *and* $\lambda(\mathcal{I}_j \cap \mathcal{P}_j) = \lambda(\mathcal{P}_j)$, $\forall j \in \{A, B\}$.

(ii) ***Total mass advertising*** *If* $c_j < q\pi_j p_j$ $\forall j \in \{A, B\}$, *then in any targeting equilibrium both firms behave as mass advertisers, i.e.,* $\lambda(\mathcal{I}_j) = 1$ $\forall j \in \{A, B\}$.

(iii) ***Coordination*** *Let* $c_j > q\pi_j p_j$ *and* $c_{-j} > q\pi_{-j}p_{-j} + (1-q)p_{-j}$. *If* $c_j < q\pi_j p_j + (1-q)p_j$, *then any non-overlapping targeting profile that satisfies* $\lambda(\mathcal{I}_j \cap \mathcal{P}_j) = \lambda(\mathcal{P}_j)$ *and* $\lambda(\mathcal{I}_A \cup \mathcal{I}_B) = 1$ *constitutes an equilibrium. If instead* $c_j > q\pi_j p_j + (1-q)p_j$, *then any non-overlapping targeting profile that satisfies* $\lambda(\mathcal{I}_A \cup \mathcal{I}_B) = 1$ *constitutes an equilibrium.*

(iv) ***Partial mass advertising*** *If* $c_j < q\pi_j p_j$ *and* $c_{-j} \in (q\pi_{-j}p_{-j}, q\pi_{-j}p_{-j} + (1-q)p_{-j})$, *then in any targeting equilibrium* $\lambda(\mathcal{I}_j) = 1, \lambda(\mathcal{I}_{-j} \cap \mathcal{P}_{-j}) = \lambda(\mathcal{P}_{-j})$ *and* $\lambda(\mathcal{I}_{-j} \cap \mathcal{P}_j) = 0$. *If instead* $c_j < q\pi_j p_j$ *and* $c_{-j} > q\pi_{-j}p_{-j} + (1-q)p_{-j}$, *then in any targeting equilibrium* $\lambda(\mathcal{I}_j) = 1$ *and* $\lambda(\mathcal{I}_{-j}) = 0$.

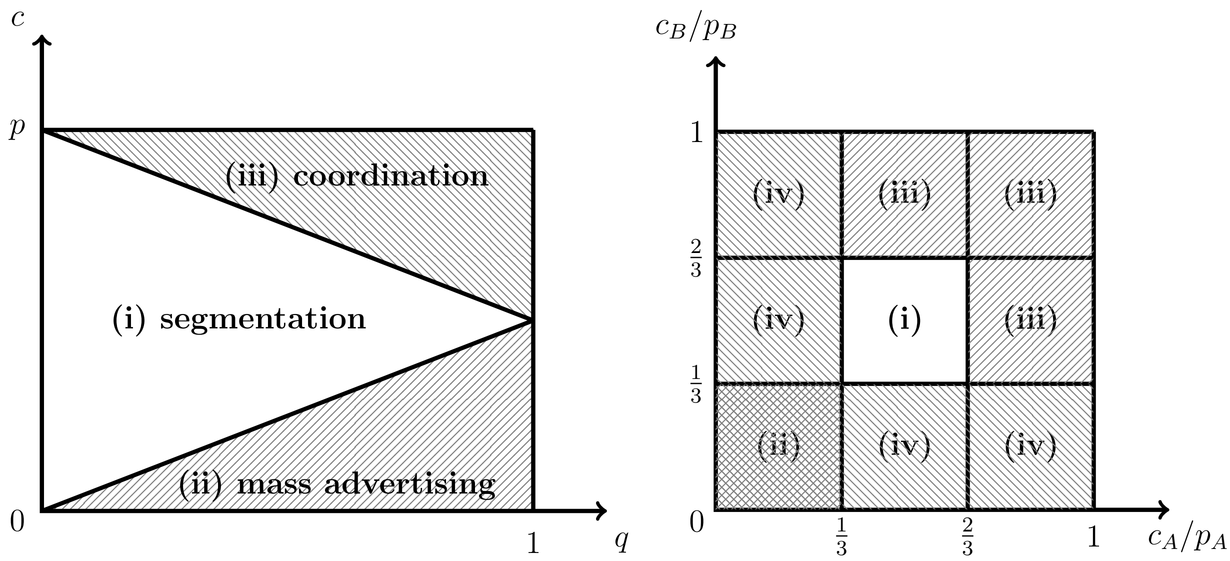
Since we allow for general Lebesgue-measurable targeting functions we never get uniqueness of equilibrium in a narrow sense, but all equilibria pertaining to cases (i), (ii) and (iv) in Theorem 1 are unique in the measure λ of targeted consumers they induce. For example, if (1) holds and the segmenting equilibrium (i) results, a natural targeting equilibrium is given by the interval strategies

$$g_A(i) = \begin{cases} 1 & \text{if } i \in [0, i_0], \\ 0 & \text{otherwise,} \end{cases} \qquad g_B(i) = \begin{cases} 1 & \text{if } i \in [i_0, 1], \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where i_0 is determined by (2). Other equilibria differ from (4) by a zero-measure set.

Theorem 1 shows that consumer inattention can have a radical effect on the nature of the targeting equilibrium. If q is sufficiently small, including the standard case where nobody is attention-constrained ($q = 0$), then the segmenting equilibrium (i) must arise. This equilibrium features non-intrusive (hence also non-overlapping) targeting strategies, and the sets of targeted consumers coincide with the respective prime segments.[20] Such a segmenting nature of equilibrium targeting strategies with the (implicit) assumption of fully attentive recipients has already been recognized by others (e.g. Iyer et al., 2005; Esteban and Hernandez, 2007; Brahim et al., 2011; Esteves and Resende, 2016). The equilibrium changes fundamentally with sufficient consumer inattention. For q large enough, the targeting equilibria are characterized by the inattention-specific equilibria (ii), (iii) or (iv), but never by (i). The strongest contrast to (i) occurs in equilibria of type (ii), showing a striking change in the nature of targeting intentions, where both firms strategically behave like *mass advertisers*, despite the availability of infinitely precise targeting possibilities.

[20]The non-intrusion property is a general result and not driven by zero-mass consumers. Further, if the set of indifferent consumers is of a positive measure, these consumers could possibly be targeted by both firms (hence there is overlap) but, by definition, targeting would not be intrusive.



(a) $c_j = c,\ p_j = p,\ \pi_j = \frac{1}{2}\ \forall j = A, B.$ (b) $\pi_A = \pi_B = \frac{1}{2}$, and $q = \frac{2}{3}$

Figure 1: Equilibrium Regimes.

Figure 1(a) illustrates the joint effects of information costs and consumer inattentiveness in the symmetric case where $c_j = c$, $p_j = p$ and $\pi_j = 1/2$, $j = A, B$. For any given $q \in (0, 1)$ the equilibrium information provision coincides with the respective consumer preferences if and only if information costs are neither too high nor too low. With either low or high information costs an inattention-specific targeting equilibrium arises, featuring mass advertising or a coordination game, where the market could be split in any arbitrary way. The range of c for which the segmenting equilibrium results decreases as q increases, and vanishes for $q = 1$.

The general reason why the targeting equilibrium changes from segmenting to mass advertising with inattentive consumers is that limited attention alters the strategic role of information provision. Besides simply informing otherwise uniformed consumers, information targeted at prime consumers works as an effective shield against business stealing by the competitor if and only if the consumer is attentive to the message; sending a message to an attentive prime consumer thus is necessary and sufficient for a successful transaction. Moreover, j would never rationally send any messages to a non-prime, attentive consumer with $-j \in X_i$ because its messages could never crowd out the ones of the superior competitor. Because business stealing is impossible if information assures attention, this disciplines the firms to restrict their targeting to the respective prime segments. The presence of inattentive consumers diminishes the protective role of information provision in a firm's targeting decision, because there is a chance that some prime consumers perceive the non-prime firm $-j$ even if $j \in X_i$. With a high degree of inattention q and a low enough information cost c_j intruding the the competitor's prime segment becomes profitable, and mass advertising becomes j's dominant strategy, meaning that the mass advertising equilibria of either type (ii) or (iv) arise.

Figure 1(b) provides a full illustration of the equilibrium regions, where we set $\pi_A = \pi_B = 1/2$ and $q = 2/3$. The figure highlights the role of the information costs. With information costs so low that (iii) does not apply (i.e., the ratios c_A/p_A and c_B/p_B are sufficiently small), any equilibrium necessarily features intrusive targeting since at least one firm mass advertises. In contrast, if information costs are high enough that (iii) applies, any equilibrium must be non-overlapping, and thus also generically intrusive. The reason is that neither firm would want its targeting strategy to overlap with the other's because its information expenditure is not covered by the expected revenue of mutually targeted consumers. There are many possible equilibria in this case, reflecting the coordination problem that firms do not know which targeting behavior the competitor adopts. From the firms' perspectives, every coordination equilibrium is Pareto efficient, and, in the symmetric case, yields the same total profit for the industry.[21] As we will see, the coordination equilibria (iii) are sensitive to the assumptions about the information cost structure. In particular, the coordination equilibria become less and the mass advertising equilibria more likely to occur, e.g., with a pay-per-click cost structure. Moreover, in section 2.2.1 we establish that if information costs are an endogenous outcome of firm competition for attention, the coordination equilibria vanish.

Summarizing, the simple baseline model reveals that the existence of a positive measure of inattentive consumers is necessary for targeting equilibria to be intrusive, and sufficient if the information costs are small enough.

[21] Although some other criteria (e.g., consumer-welfare-maximizing) could allow to select a subset of the coordination equilibria, this does not imply that any resulting predictions of such a selection in the high-information-cost case will be robust.

2.1.1 Discussion

In this section, we discuss the main implications of LA for market shares and consumer welfare on the basis of our simple framework. In addition, we examine whether an information tax levied on attention-seekers could improve informational efficiency. We further consider the consequences of an alternative, relevant assumption on information costs (pay-as-click).

Market shares Theorem 1 implies that comparative advantages in attention seeking may replace preferences or the degree of product differentiation as the main determinants of a firm's market share. To see this, fix c_j, p_j, π_j for $j = A, B$ such that $c_j < \pi_j p_j$. As long as $q < \frac{c_j}{\pi_j p_j}$ for $j = A, B$ then, by Theorem 1 (i), equilibrium targeting, market shares $m_j = \lambda(P_j)$ and profits $\Pi_j = (p_j - c_j)m_j$ depend only on the size of the prime segment, and are invariant to the degree of inattention q. This changes drastically if instead $q > \frac{c_j}{\pi_j p_j}$ for $j = A, B$. Then, by (ii), equilibrium targeting is independent of $\mathcal{P}_j$, and thus independent from what defines prime segments. In addition, firm j's market share $m_j = q\pi_j + (1-q)\lambda(\mathcal{P}_j)$ and profit $\Pi_j = m_j p_j - c_j$ are increasing in q if and only if $\pi_j \geq \lambda(\mathcal{P}_j)$. This reveals that an increase in q redistributes some market share (and profit) to the firm that has a comparative advantage in attention-seeking, because if $\pi_{j'} \neq \lambda(\mathcal{P}_{j'})$ for some $j' \in \{A, B\}$ then necessarily $\pi_j > \lambda(\mathcal{P}_j)$ and $\pi_{-j} < \lambda(\mathcal{P}_{-j})$. Finally, note from (iv) that being a comparably strong attention-seeker may be the only way how an otherwise inferior firm can gain a large market share, while failing to attract attention may annihilate any comparative advantage from offering a superior product.[22] This further implies a possible asymmetry between the firms in their desire for inattention in a market, which is elaborated in Appendix A.3.

Welfare Because information is costly, efficiency requires that each firm exclusively targets its prime segment.[23] In particular, any targeting profile featuring intrusion is inefficient. A central normative implication of Theorem 1 is that the targeting equilibrium is generically efficient if and only if the degree of inattention q in the market is low enough. To see this, note from Theorem

[22]Indeed, if $\mathcal{P}_j = \emptyset$ but $q > c_j/(\pi_j p_j)$ then $m_j \geq q\pi_j$, where even $m_j = 1$ if $\pi_{-j} < (c_j/p_j - (1-q))/q$. Hence if an otherwise completely inferior firm can find a way to make its product significantly more salient, it can drive its competitor out of business.

[23]A different type of inefficiency occurs if some consumers remain totally uninformed (Butters, 1977; Grossman and Shapiro, 1984), which does not arise in the equilibria of our model.

1 that the segmenting equilibrium in (i) is efficient, while the inattention-specific equilibria (ii) and (iv) are inefficient as they involve intrusion, and the coordination equilibria in (iii) are generically inefficient.[24] It then suffices to realize that for $q < \min\{\frac{c_j}{\pi_j p_j}, \frac{p_j - c_j}{p_j(1-\pi_j)}\}$, $j = A, B$, the efficient segmenting equilibrium results, while if $q > \min\{\frac{c_j}{\pi_j p_j}, \frac{p_j - c_j}{p_j(1-\pi_j)}\}$ for some $j = A, B$ a (generically) inefficient attention equilibrium of type (ii)-(iv) results. At the extremes, the equilibrium is always efficient with only attentive consumers ($q = 0$) and always (generically) inefficient with only inattentive consumers ($q = 1$).

The line structure of the model allows to assess the magnitude of the *consumer* welfare loss associated with LA. In the following, we compare consumer welfare in the two polar cases $q = 0$ and $q = 1$, where we let $\mathcal{V}_A = \mathcal{V}_B$ and, for $q = 1$, $c_j < \pi_j p_j \ \forall j = A, B$, such that mutual mass advertising results. In each case, we measure consumer welfare by the (expected) aggregated transportation cost. With attentive consumers, this cost is equal to $T_{UA} \equiv t\left[\sum_{j=A,B} \int_{i \in \mathcal{P}_j} |i - x_j| di\right]$. However, if consumers are attention-constrained, their purchasing decisions depend only on the salience parameters (π_A, π_B), and the resulting aggregate transportation cost is given by $T_{LA} \equiv t\left[\sum_{j=A,B} \pi_j \left(\frac{1}{2} - x_j(1 - x_j)\right)\right]$. Normalizing by marginal transportation cost t, the welfare loss due to LA then is

$$\frac{T_{LA} - T_{UA}}{t} = (x_B - x_A)\left(\pi_A\left(1 - \frac{1}{4}x_A - \frac{3}{4}x_B\right) + \pi_B\left(\frac{3}{4}x_A + \frac{1}{4}x_B\right)\right). \tag{5}$$

While LA always implies a welfare loss ($T_{LA} > T_{UA}$), this loss becomes small if products are close substitutes ($x_B \approx x_A$), because with weak differentiation it is not important which alternative is consumed. Increasing differentiation tends to increase the welfare loss. Especially, with symmetric firm locations, i.e. $x_A = 1 - x_B$, the welfare loss is independent of the salience parameters π_A, π_B (a consequence of symmetry) and increases in the distance between x_A and x_B.[25] The latter holds because average disutility of choosing the wrong firm increases with more polarized firms.

[24]Note that in case of (iii) the efficient equilibrium would never arise if firms choose targeting strategies sequentially rather than simultaneously given the coordination nature of the equilibrium. The first-mover would target the entire market while the follower shuts down.

[25]For asymmetric locations, the relation is more subtle because a change of x_j has non-monotone effects on T_{LA} and T_{UA}. The maximal welfare loss is 1/3 and occurs if $x_A = 1/3$, $x_B = 1$ and $\pi_A \approx 0$ or $x_A = 0$, $x_B = 2/3$ and $\pi_B \approx 0$.

While the possibility of a mismatch is not an issue for an attentive consumer, she may still suffer in the mass advertising equilibrium if each message from the firms causes a small nuisance to her.[26] Therefore, the presence of inattentive consumers can indirectly harm the attentive ones by alluring firms to fill the mailbox of every consumer, leading to a search externality between inattentive and attentive consumers (Armstrong, 2015). More generally, a small increase in q may discontinuously alter the equilibrium from segmenting to mass advertising, thereby imposing a large negative welfare effect on all inattentive consumers and, in presence of a nuisance cost, also on attentive consumers.

Information tax In view of the inefficiencies associated with LA, a natural question to ask is whether an information tax could improve welfare by changing equilibrium from mass advertising to the non-intrusive, efficient equilibrium. The answer depends on the circumstances. Consider the case where a tax (or a subsidy) $\tau \in \mathbb{R}$ is levied on each unit of information issued by a firm. The effective unit cost of information for firm j then becomes $c_j + \tau$. Some years ago, the French government considered the introduction of a similar advertising tax on the sponsored links of search engines. Because the tax shifts marginal costs, we can use Theorem 1 to investigate the tax effects, replacing "c_j" by "$c_j + \tau$" in the theorem.

Corollary 1 (Information tax) *The informationally efficient equilibrium can be induced by a tax or a subsidy* $\tau \in \mathbb{R}$ *if and only if*

$$c_A - c_B \in \left(\underline{c}_A - \bar{c}_B, \bar{c}_A - \underline{c}_B\right),$$

where $\underline{c}_j = q\pi_j p_j$ *and* $\bar{c}_j = \underline{c}_j + (1-q)p_j$ $\forall j \in \{A, B\}$.

To illustrate the corollary, we first construct a situation where an appropriate tax can induce the informationally efficient equilibrium. Consider the symmetric case where $c_j = c$, $p_j = p$

[26] Further implications of nuisance costs are explored in the context of ad avoidance (see section 3).

and $\pi_j = 1/2$. Further, suppose that $c < pq/2$, such that without the tax the mass advertising equilibrium (ii) results. Since the condition in Corollary 1 is satisfied, a tax τ exists such that the informationally efficient equilibrium is induced. As Figure 2 shows, a wrongly calibrated tax

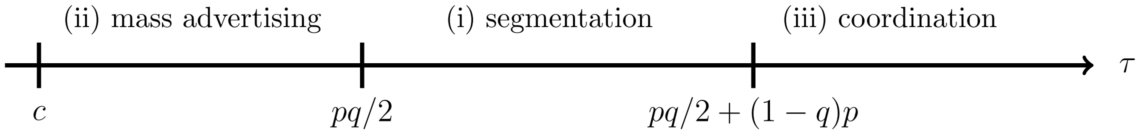


Figure 2: Tax effects with perfectly symmetric firms

can either have no or an adverse effect. Given that $c < pq/2$, the equilibrium is invariant to τ if τ is too small ($c + \tau < pq/2$), or it changes to a coordination equilibrium (iii) if τ is too large. Generally, the range of τ for which the efficient equilibrium emerges (if at all) is decreasing in q for *any* parameter constellation, and vanishes if $q = 1$. The condition in Corollary 1 is violated with asymmetric firms if the attention-dominant firm has a sufficiently strong attention-price advantage. For example, if firm A has such an advantage ($\pi_A > \pi_B$ and $p_A \geq p_B$), the existence condition reduces to $q\pi_A p_A \leq (q\pi_B + 1 - q)p_B$ for the case of symmetric costs. This inequality is more likely to be violated, for any given $q > 0$, the larger $\pi_A p_A$ becomes. Finally, we show in section 2.2.1 that the targeting behavior is entirely invariant to an information tax if targeting costs are determined by the salience competition between the firms.

Pay-per-click cost structure So far, we have considered the case where firms incur an information cost per targeted consumer independent of whether they end up with a successful transaction. This is a standard cost structure in the advertising literature, and it is sometimes called a pay-per-impression pricing scheme in the marketing jargon. The fact that performance-based pricing schemes, to which pay-per-click prominently belongs, have gained on importance among the pricing models implemented by publishers (Hu et al., 2015) makes it important to ask whether a pay-per-click cost structure could change the core predictions of our analysis. The answer to this question is no, and pay-per-click costs make inattention-specific equilibria even more likely to arise.

Suppose that firm j incurs a unit cost c_j for its targeted information if and only if it receives a click by a consumer. To make the pay-per-click pricing scheme meaningful in the context of LA, we assume that attention-constrained consumers only click on a single ad targeted at them, while consumers with sufficient attention capacities click on, and thus pay attention to, all the ads targeted at them.[27] Hence, the pay-per-click cost structure effectively matters only for attention-constrained consumers, where firm j pays c_j if and only if it receives the consumer's exclusive attention, and thus also makes a sale. It turns out that the pay-per-click cost structure does not qualitatively change how LA can affect the equilibrium targeting behavior.

Corollary 2 (Pay-per-click cost structure) *Suppose that firms face the above pay-per-click cost structure, $p_j > c_j > 0$ and $\pi_j \in (0, 1)$ $\forall j = A, B$. Then Theorem 1 applies, where "c_j" in (i) - (iv) is replaced by "$c_j(q\pi_j + (1 - q))$", implying that the inefficient coordination equilibria in (iii) cannot emerge.*

Corollary 2 shows that pay-per-click costs make the efficient segmenting equilibria of type (i) less likely and the mass advertising equilibria of type (ii) more likely. Intuitively, with the pay-per-click cost structure the expected cost from mutually targeted consumers is lower (since $q\pi_j + 1 - q \leq 1$), making it more profitable for a firm to target its non-prime consumers.[28] In addition, targeting all consumers in its prime segment becomes a dominant action for each firm j, regardless of its cost c_j. This is because whenever a consumer in $\mathcal{P}_j$ chooses firm $-j$, firm j incurs no costs from targeting that consumer thanks to the pay-per-click cost structure. As a result, the coordination equilibria of type (iii) completely vanish.

[27] If we alternatively assumed that attention-constrained consumers click on all ads they see but remember only one firm when making their consumption decisions, the relevant information cost and the strategic analysis would be identical to our original model.

[28] The same qualitative impact will be obtained if we instead assume that the attentive consumers will know, based on ads, which firm is the relevant one and will click on only that one, whereas inattentive consumers will click on all ads (but can remember only one of them).

2.2 Extensions

We extend the baseline model in several directions. Firstly, we endogenize perception probabilities (π_A, π_B) and (marginal) targeting costs (c_A, c_B), by introducing salience competition between attention-seekers. Secondly, we introduce price competition to the targeting framework. Thirdly, we consider imperfect targeting abilities as a consequence of incomplete marketing data. In all these variations, we find LA to have the same core effects on equilibrium information dissemination as in the baseline model, while we gain additional insights on the determinants of effective targeting costs, on firm-side information acquisition motives and on strategic pricing.

2.2.1 Salience competition

There is a lot of evidence suggesting that the attention which firms attract from inattentive consumers depends on the degree of conspicuousness or salience of their ads relative to each other.[29] Such efforts to attract or retain attention can take on many forms, for instance advertising over multiple channels (emails, social network, phone calls, etc.) to the same consumer or by retargeting consumers who showed previous interest in a product. How does the ability to manipulate the relative salience of their messages affect which consumers the information-senders seek to target? To address this question we extend the baseline model by incorporating salience competition as in Hefti (2015) for mutually targeted consumers. Each firm must decide on how much to invest into the salience of its messages, besides choosing its targeting strategy. Salience competition determines the effective targeting costs, and we show that the resulting strategic competition for attention implies that the coordination equilibria (iii) vanish, and mass advertising becomes the sole equilibrium prediction.

Formally, each firm j needs to choose a *salience function* $s_j : [0, 1] \to \mathbb{R}_+$, which specifies the salience level $s_j(i)$ of the information targeted at a consumer i, besides the decision whom to target. Thus, firm j's strategy now is a pair (g_j, s_j). The chosen salience levels endogenize the perception probabilities (π_{Ai}, π_{Bi}) by the following principle. If an inattentive consumer i is targeted only by a single firm, this firm will always be perceived independent of its chosen

[29] See Hefti and Heinke (2015) and the references therein for examples.

salience level; if the consumer is targeted by both firms, there is a contest for attention and the perception chances are determined by *relative salience*:

$$\pi_{ji} = \begin{cases} \frac{s_j(i)}{s_A(i)+s_B(i)} & \text{if } s_A(i) + s_B(i) > 0, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{6}$$

Salience-generating activities have the effect of increasing own while decreasing the competitor's perception chances. Increasing own salience is costly. In particular, targeting a message to a consumer at a salience level s requires the firm to incur a cost of $h(s)$, where $h : \mathbb{R}_+ \to \mathbb{R}_+$ is a convex and strictly increasing C^2-function with $h(0) = 0$ and $\lim_{s \to 0} h'(s) < \infty$. Accordingly, for a given information campaign (g_j, s_j), firm j's total information expenditure is equal to $\int_{[0,1]} h(s_j(i))di$. It follows that the (marginal) information costs are now endogenous and depend on the chosen salience levels.[30]

Because competition for attention emerges only for attention-constrained consumers, we let $R_i = 1\ \forall i \in [0, 1]$ as a simplifying assumption.[31] Further, we set $V_j = V$ and take $p_j = p \in (0, V - t\max\{1 - x_A, x_B\}]$ as exogenously given, $\forall j \in \{A, B\}$.[32] To derive the targeting-salience equilibrium, we first note that in equilibrium the salience levels are pinned down by the targeting profile (g_A, g_B). Especially, it is optimal for firm j to set $s_j(i) = 0$ if either $g_j(i) = 0$ or $g_j(i) = 1$ but $g_{-j}(i) = 0$; rational firms invest into salience only for mutually targeted consumers. It directly follows that there cannot be a non-zero set of untargeted consumers in equilibrium, consistent with the baseline model. Now, consider a consumer $i \in \mathcal{I}_j$ that is targeted by firm $-j$ at salience level $s_{-j}(i) > 0$. In this case, firm j's optimal salience level for consumer i obeys the first-order condition[33]

$$p\frac{s_{-j}(i)}{(s_j(i) + s_{-j}(i))^2} = h'(s_j(i)). \tag{7}$$

In a symmetric equilibrium, $s_A(i) = s_B(i) = s$, and (7) reduces to $p = 4h'(s)s$, which admits

[30]Recall that in the baseline model, firm j's total information cost is given by $c_j \int_{[0,1]} g_j(i)di$. This can be viewed as a special case of the current model where firm j is exogenously given the following salience function: $s_j(i) = 0$ if $g_j(i) = 0$ and $s_j(i) = h^{-1}(c_j)$ otherwise.

[31]In terms of targeting strategies, mass advertising would remain the unique prediction of the salience-targeting model whenever q is large enough, while the segmenting equilibrium were to result if q is small.

[32]The main conclusion from this section would also apply if prices were endogenized as in section 2.2.2.

[33]To avoid measurability problems, we restrict attention to functions $s_j(\cdot)$ that respect (7) for all $i \in \mathcal{I}_j$.

a unique solution $s^* > 0$ given the assumptions on $h(\cdot)$. Hence, firm j's expected payoff from targeting such a consumer i is given by

$$\Pi_j(i) = \frac{p}{2} - h(s^*) = 2h'(s^*)s^* - h(s^*) \geq 2h(s^*) - h(s^*) = h(s^*) > 0, \tag{8}$$

where the first inequality follows from the convexity of $h(\cdot)$ and $h(0) = 0$. Consequently, both firms would indeed find it optimal to target the entire market while setting a uniform salience level s^* for their messages. Because no asymmetric salience equilibrium can exist, the symmetric equilibrium is in fact the unique equilibrium:

Proposition 1 *There exists an essentially unique equilibrium, and both firms behave as mass advertisers, i.e.* $\lambda(\mathcal{I}_A) = \lambda(\mathcal{I}_B) = 1$*, and set the same salience level* $s^* > 0$ *for all mutually targeted consumers, where* s^* *uniquely solves* $p = 4h'(s^*)s^*$*.*

Modern advertising firms voice substantial concerns about whether their messages are even registered by consumers.[34] Consequently, a sizable sum of money is invested to remain "top of the mind", despite the availability of sophisticated marketing data and the fact that products may already be familiar to many consumers (Iyer et al., 2005). While such an empirical regularity is hard to rationalize with perfectly attentive consumers – such salience competition would not occur even with imperfect marketing data about consumer preferences – Proposition 1 establishes that it follows from the presence of inattentive consumers, where firms compete for consumer attention jointly with behaving as mass advertisers. Furthermore, Proposition 1 implies that an information tax in the spirit of Corollary 1 cannot change the mass advertising behavior. The only effect of such a tax were to scale down the equilibrium salience levels without affecting the relative perception chances, leaving the targeting incentives likewise unchanged. Finally, we show in Appendix A.5 that mass advertising is the only equilibrium prediction in the salience-targeting framework even if the salience effects interact with consumer preferences.

[34] See, e.g., "Invisible ads, phantom readers", *The Economist*, Mar 2016.

2.2.2 Price competition

In this section we enrich the baseline targeting model with strategic pricing. Specifically, we consider a two-stage complete information game, where each firm first sets its price and then decides which subset of consumers to inform. Many circumstances in a modern, digital economy indeed suggest that firms have knowledge about each others pricing strategy, prior to launching their advertising campaigns. For example, while iPhone7 was not yet available at the time the current paper was firstly drafted, its price has already been leaked by several websites,[35] and such information would be almost surely noticed by the competitors of Apple (if they didn't know it already). A related point is the common observation that once prices are made, firms are reluctant to change them, which is a well-known result in presence of menu costs (see, e.g., Golosov and Lucas, 2007). Likewise, the fact that most advertising has become digital, predominantly featuring real-time auctions for ad placement in digital ad space, strongly suggests that (targeted) advertising can be adjusted nearly instantaneously.[36]

Formally, each firm's strategy is a pair $p_j \in \mathbb{R}_+$ and $g_j : [0,1] \times \mathbb{R}^2_+ \to \{0,1\}$, where $g_j(i, p_A, p_B) = 1$ iff consumer i is targeted by firm j given that first-stage prices are p_A and p_B. The firms' targeting strategies are now contingent on the prices that they set. We assume that $V > c + 2t$, which assures that a monopolist located at any $x \in [0,1]$ always chooses to cover the entire market at a price $p = V - t \cdot \max\{x, 1-x\}$. As a simplifying tie-breaking rule, we also assume that whenever firms are indifferent about targeting a non-zero measure set of consumers they choose to target that set.[37] To see most clearly the effect of LA with endogenous pricing, we derive and compare the Subgame Perfect Equilibria (SPE) that arise in the two polar cases where $q = 0$ and $q = 1$.

[35] See, e.g., http://bgr.com/2016/06/30/this-is-the-iphone-7-leak-weve-been-waiting-for.

[36] See the "AdEX Benchmark H1 2017 Study" published by IAB Europe. A related video illustration is available at https://goo.gl/YcUjJy. This is different from non-targeted advertising or classical non-digital advertising, such as the design of a TV spot or a poster, which cannot be easily adjusted in the short run, and which would suggest the opposite timing of the game.

[37] This tie-breaking rule is only needed for the uniqueness (but not existence) of the SPE in case (i) of Proposition 3, and it would not be needed at all if $c \leq t$ is satisfied (see the discussion following the proof of Proposition 3).

Unlimited attention We first characterize the unique SPE outcome in the benchmark case of unconstrained attention. For simplicity, we focus entirely on equilibria with interval targeting strategies of type (4).

Proposition 2 *Suppose that* $R_i > 1 \quad \forall i \in [0,1]$. *In any SPE, the targeting profile is non-intrusive, with consumers located in* $[0, i_0^*]$ *and* $[i_0^*, 1]$ *being targeted by firm* A *and* B*, respectively, where* $i_0^* = \frac{1}{3} + \frac{x_A + x_B}{6}$*, and*

$$p_A^* = \left(\frac{2}{3} + \frac{x_A + x_B}{3}\right) t + c, \quad \Pi_A = \frac{t(2 + x_A + x_B)^2}{18},$$
$$p_B^* = \left(\frac{4}{3} - \frac{x_A + x_B}{3}\right) t + c, \quad \Pi_B = \frac{t(4 - x_A - x_B)^2}{18}.$$

Proposition 2 establishes that with unlimited attention only a non-intrusive (and thus non-overlapping) targeting equilibrium is obtained, which is the same type of targeting behavior as identified by the baseline model (Theorem 1 (i)).[38] Moreover, information costs are entirely passed through to consumers, and a decreasing willingness-to-substitute in the consumer population as captured by a higher t increases firm prices and profits. While the proof is not trivial, the main intuition is quite simple. A key step is to note that for any given pricing strategy (p_A, p_B) there is a marginal consumer $i_0 \in [0,1]$ such that $U_i(A) > U_i(B)$ if $i < i_0$ and $U_i(A) < U_i(B)$ if $i > i_0$. Optimality then requires that the firms' targeting behavior must match this price-induced segmentation of the market. In particular, targeting either "below" or "above" i_0 can never be part of an SPE, because either the firm would forfeit additional revenue from infra-marginal consumers or it would bear unnecessary information costs from non-captive super-marginal consumers.

An interesting side observation is that prices and profits in the SPE correspond exactly to their counterparts if consumers had *complete* ex-ante information and firms could compete only

[38] Roy (2000) studies a targeting-pricing game with *homogeneous* goods and the reversed timing. Similar to us, he finds that the targeting strategy profile arises in any SPE must be non-overlapping. However, in his model firms face a coordination problem since its SPE outcome is not unique except when information costs converge to zero.

in prices, where c then had the conventional interpretation as production costs. In this sense, Proposition 2 shows that firms have limited abilities to "make the price", despite the fact that the market is informationally partitioned and consumers become aware of only one firm. In particular, deviating to a higher price (e.g. the monopoly price) would not be profitable for any firm, since it would induce its competitor to subsequently deviate to a targeting strategy that will reduce its market share for sure.

Limited attention The next proposition shows that with LA and endogenous pricing essentially the same types of targeting behavior as in the inattention-specific equilibria of the baseline model (Theorem 1 (ii)-(iv)) results.

Proposition 3 *Suppose that* $R_i = 1 \; \forall i \in [0, 1]$.

(i) *If* $c < \pi_j(V - t(1 + \max\{x_j, 1 - x_j\}) \; \forall j \in \{A, B\}$, *then in any SPE both firms behave as mass advertisers, i.e.* $\lambda(\mathcal{I}_j) = 1 \; \forall j \in \{A, B\}$, *and set prices* $p_j^* = V - t \max\{x_j, 1 - x_j\}$ *and* $\Pi_j = \pi_j p_j^* - c \; \forall j \in \{A, B\}$.

(ii) *If* $c > \max\{\pi_A V, \pi_B V\}$, *then in any SPE the targeting profile is non-overlapping,* $p_j^* = \inf_{i \in \mathcal{I}_j}\{V - t|i - x_j|\}$ *and* $\Pi_j = (p_j^* - c)\lambda(\mathcal{I}_j) \; \forall j \in \{A, B\}$. *In addition, any non-overlapping targeting profile together with* $p_j^* = \inf_{i \in \mathcal{I}_j}\{V - t|i - x_j|\}$, $j = A, B$, *can be supported as part of a SPE.*

(iii) *If* $\pi_{-j}V < c < \pi_j(V - t \max\{x_j, 1 - x_j\})$, *then in any SPE* $\lambda(\mathcal{I}_j) = 1$, $\lambda(\mathcal{I}_{-j}) = 0$, $p_j^* = V - t \max\{x_j, 1 - x_j\}$, $\Pi_j = p_j^* - c$ *and* $\Pi_{-j} = 0$.

According to Proposition 3, mass advertising is the unique equilibrium prediction once information costs are sufficiently low (i), a coordination problem emerges with mutually high information costs (ii), and strong unilateral attentional advantages may allow a firm to drive her competitor out of the market (iii). The intuition for these results strongly parallel their simpler counterparts in section 2.1. For example, if the parametric assumption in (i) is satisfied, then each firm prices to the entire market as an effective monopolist (with a location-depending monopoly price) accompanied by mass advertising. This is optimal because, with a sufficiently low information cost, the inability of firms to shield their prime segments from business stealing once again makes intrusive targeting profitable.

The qualitative effects of locations x_A, x_B for targeting strategies and equilibrium payoffs with and without LA are similar to those from the baseline model. Likewise, firm market shares under LA reflect exclusively the comparative advantage in attention seeking in a mass advertising equilibrium. LA may have an additional implication on how certain consumer characteristics influence equilibrium payoffs with endogenous pricing. Most notably, equilibrium prices and profits increase in the taste parameter t with attentive, but decrease in t with inattentive consumers. The reason is that t determines both the consumers' willingness-to-substitute and the decisions on whether or not to acquire any product. With attentive consumers and the full coverage assumption we made, the second channel is obsolete in the competitive equilibrium, implying that a weaker willingness-to-substitute (higher t) leads to higher prices and profits. On the contrary, with LA it is only the threat of exiting consumers that matters for firms. A higher t (or similarly a lower V) increases this threat, forcing firms to set lower prices. This also explains why the mass advertising scenario (i) of Proposition 3 becomes more likely if V_j increases, t decreases or firms are located towards the midpoint of $[0, 1]$. In all cases the equilibrium price increases, which makes intrusive targeting more profitable.

Regarding welfare, the effects of LA identified in the baseline model remain present in the model with price competition. In particular, with LA the equilibrium is inefficient if mass advertising occurs, and generically inefficient in the case of coordination equilibria, where the former is more likely to occur with low information costs. Without LA, there is no wasteful information provision in equilibrium as targeting profiles never overlap. Further, the equilibrium is always efficient with symmetric firm locations ($x_A = 1 - x_B$), as then always $i_0^* = 1/2$, and constrained efficient with asymmetric locations, because price competition results in a suboptimal point of segmentation. Regarding consumer welfare, the welfare loss due to LA in terms of transportation costs again is given by (5) if parameters are such that with LA the mass advertising equilibrium occurs. With endogenous pricing, LA implies an additional consumer welfare loss as consumers are charged comparably higher prices. This can best be seen for symmetric firm locations (which implies that $x_B > 1/2$), where the common equilibrium price with LA is $p_{LA} = V - tx_B$ (Proposition 3 (i)), and $p_{UA} = t + c$ with unlimited attention (Proposition 2), and $p_{LA} > p_{UA}$ if $V > 2c + t$ which holds by the full coverage assumption.

A final observation is that with LA and endogenous pricing firms can be harmed by *lower* information costs. To see this, suppose $x_A = 0, x_B = 1, \pi_A = \pi_B = 1/2$ and $c > V/2$. According to Proposition 3, there exists an equilibrium in which each firm sets a price $p = V - t/2$ and subsequently targets a different half of the consumers in the market. Now, if the information cost decreases to $\hat{c} < V/2 - t$, the two firms will set a new price $\hat{p} = V - t$ and target every consumer in the market in the new equilibrium, but will still end up with sharing the market equally. Consequently, the equilibrium profits of firm j changes from $\Pi_j = (V - t/2 - c)/2$ to $\hat{\Pi}_j = (V - t)/2 - \hat{c}$. It is easy to see that $\hat{\Pi}_j < \Pi_j$ if $\hat{c} > (2c - t)/4$. Intuitively, a high targeting cost makes the firms able to credibly commit not to penetrate their opponent's market.

2.2.3 Imperfect marketing data

In this section we relax the assumption that firms ex ante have perfect data about consumer tastes. In reality, firms can acquire consumer data through several channels.[39] This further raises the question if and how LA could influence this information gathering process in the first place. The two main results of this section are i) LA and information costs affect the targeting behavior in essentially the same way as in the baseline model and ii) the incentives to acquire more precise data depend on consumer attention.

We consider a continuum of consumers indexed by $i \in [0, 1]$. Consistent with the baseline model, each consumer has a strict preference $\succ_i$ defined over two firms A and B, where $A \succ_i B$ if $i \in \mathcal{P}_A = [0, 1/2]$ and $B \succ_i A$ if $i \in \mathcal{P}_B = (1/2, 1]$.[40] Contrary to the baseline, each firm may

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[39] In most cases, firms collect consumer data either from a direct firm-customer relationship ("First-Party" data) or by acquiring data from specialized companies ("Third-Party" data). Third-Party data usually is collected on the Internet using digital cookies, web beacons or e-tags without consumers being aware of them (see "Advertising and technology", *The Economist*, Sep 2014, Special Report).

[40] We thus ignore the non-competitive case, where some consumers have no valuation for one or both of the products. This is not a serious omission for our purpose, because a firm is harmed by costly targeting a consumer with no interest in its products, independent of consumer attention, meaning that data revealing such

have imperfect knowledge about consumer preferences, capturing a possible limitation in the precision of the marketing data. We assume that the marketing data of each firm j comprises all relevant consumers and takes on the form of a simple signal $z_{ji} \in \{0, 1\}$ for each consumer $i \in [0, 1]$. Signals are independent across firms and consumers, and the signal structure is common knowledge. Specifically, we suppose that if $j \succ_i -j$, then $z_{ji} = 1$ with probability 1; if $-j \succ_i j$, then $z_{ji} = 1$ with probability $\alpha_j \in [0, 1]$ and $z_{ji} = 0$ with probability $1 - \alpha_j$, and . This means that the marketing data qualitatively reflects the true preference distribution in such that it never fails to include a firm's prime consumers, but it may exacerbate the preferences for some consumers. It follows that whenever $z_{ji} = 0$, j (correctly) infers that $i \in \mathcal{P}_{-j}$, and accordingly $z_{-ji} = 1$. Thus, α_j yields a measure of j's data quality, where a small value of α_j implies that firm j becomes more likely to correctly separate non-prime from prime consumers. For $\alpha_j = 0$ the model collapses to the baseline case, while $\alpha_j = 1$ means that j has no clue about consumer tastes.

Given its marketing data, each firm decides which consumers to target similar to the baseline model. As before, the targeting profile (g_A, g_B) determines the consumers' information sets, and each consumer forms an attention set that depends on her attention capacity. Consumer i acquires her best perceived product, and each firm j earns an exogenous revenue $p_j > 0$ for every successful transaction.

Unlimited attention We first study how the marketing data were to affect the targeting equilibrium with attentive consumers. Let $\mathcal{P}_j^* = \{i \in [0, 1] : z_{ji} = 1\}$ denote j's prime segment as *indicated by the data*. Note that $\mathcal{P}_j \subseteq \mathcal{P}_j^*$ $\forall j \in \{A, B\}$ and $\lambda(\mathcal{P}_A^* \cup \mathcal{P}_B^*) = 1$. In view of the general understanding that modern media have lowered information costs, we restrict the analysis to the case of cheap information provision.

Proposition 4 *Suppose that $R_i > 1$ $\forall i \in [0, 1]$. If $c_j < p_j/(1 + \alpha_j)$ $\forall j \in \{A, B\}$, then in any targeting equilibrium each firm only targets its indicated prime segment, i.e. $\lambda(\mathcal{I}_j \cap \mathcal{P}_j^*) = \lambda(\mathcal{P}_j^*)$*

consumers is similarly valuable independent of consumer attention.

and $\lambda(\mathcal{I}_j \setminus \mathcal{P}_j^*) = 0$*, and equilibrium profits are* $\Pi_j = p_j\lambda(\mathcal{P}_j) - c_j\lambda(\mathcal{P}_j^*)$*,* $j = A, B$*.*

In the equilibrium identified by Proposition 4, a firm will target a consumer if and only if its marketing data indicates that this consumer belongs to its prime segment. This follows because, given the false-positive nature of the data, j infers that $g_{-j}(i) = 1$ and $i \notin \mathcal{P}_j$ for any consumer with $z_{ji} = 0$. Because all consumers are assumed to be attentive, it can never be profitable to send a message to such a consumer; likewise, given that $p_j > (1+\alpha_j)c_j$, it is always a dominant action to inform any i with $z_{ji} = 1$.

Proposition 4 reveals that, different to the baseline, the equilibrium targeting profile is overlapping (and intrusive) as a consequence of imprecise consumer data, despite fully attentive consumers. In equilibrium, a fraction $(\alpha_A + \alpha_B)/2$ of consumers will be (inefficiently) informed by both firms. The crucial difference to the targeting motive induced by LA as identified in the baseline model is that with imperfect data but unlimited attention the resulting equilibrium intrusion is entirely driven by imprecise information about the market and not by a *strategic desire* to invade the competitor's prime segment. Thus, the baseline result about equilibrium targeting with attentive consumers is robust to imperfect marketing data in the sense that no firm seeks to intrude the prime segment of the other firm as indicated by their data. Any improvement in the data decreases the overlap, expressing the strategic intent to informationally partition the market. This observation has direct implications for a firm's information acquisition motives. In the unique targeting equilibrium, the number of jointly targeted consumers decreases as the quality of either firm's marketing data increases, and gradually approaches a zero-measure set as $\alpha_j \to 0$ $\forall j \in \{A, B\}$. For the same reason, each firm j's expected profit is strictly decreasing in α_j, assuring that firms have an incentive to acquire more precise data.[41]

Limited attention We now reconsider the above problem with inattentive consumers. As in the baseline model, R_i is iid over the population, where q is the measure of inattentive consumers. The following result shows that, independent of the data quality, mass advertising

[41] Our results in section 3 imply that the benefits of better firm information were even more pronounced in presence of nuisance costs. The intuition is that rational consumers will anticipate that it they are less likely to receive ads from both firms with better marketing data and, as a consequence, they are less inclined to use ad blockers. In such a case, an unilaterally better informed firm exerts a positive externality on the other firm.

is the unique equilibrium prediction by the same low cost condition identified in Theorem 1 (ii).

Proposition 5 *Suppose that* $\forall i \in [0,1]$, $R_i = 1$ *with probability* $q \in (0,1]$ *and* $R_i = 2$ *otherwise. If* $c_j < q\pi_j p_j \ \forall j \in \{A, B\}$, *then in any targeting equilibrium both firms behave as mass advertisers, i.e.* $\lambda(\mathcal{I}_A) = (\mathcal{I}_B) = 1$, *and* $\Pi_j^* = p_j \lambda(\mathcal{P}_j) - c_j \ \forall j \in \{A, B\}$.

The intuition is that the firms' concern about consumer preferences is diluted by the presence of inattentive consumers. With sufficiently low information costs, each firm is incentivized to mass advertise as it hopes to capture enough consumer attention independent of how precise its data is. This means that more precise marketing data is ineffective in reducing the targeting overlap, and hence cannot increase equilibrium profits. In contrast to the case of unlimited attention, this shows that no firm has an incentive to acquire better marketing data, nor could consumers benefit from revealing their tastes to the firms. The last insight is even reinforced if consumers have privacy concerns and the ability to use protective ad blockers, which is the core topic of our final section.

3 Information Blocking and Limited Attention

In recent years, the advertising industry has been confronted with a steeply increasing usage of ad blocking tools, jointly with consumers often expressing both a sentiment of ad overload and privacy concerns regarding their data. As targeting abilities have likewise improved, a certain surprise expressed within the ad industry about this development is understandable from a standard targeting model. According to our previous results on attentive consumers, better marketing data should have the effect that consumers more easily find their needs as markets provide better targeted information to them (Proposition 4), or lower information costs should yield more competitive prices (Proposition 2). More importantly, the segmenting nature of the targeting behavior implies that rational consumers may be willing to reveal their tastes once they understand that the market forces firms to use this information to the best of the consumer's interest, making privacy concerns less pertinent.

In this section we show that by accounting for LA the otherwise standard targeting model indeed predicts mass advertising to occur jointly with increased information blocking and concerned consumers. For this purpose, we enrich the demand side of the baseline model by introducing the consumer-side decision to block the information they may receive. The effects of LA on targeting and consumer blocking can be made most evident in the stylized case where firms have perfect targeting abilities, which we assume first, but later relax in section 3.2.

Consider the baseline model, where we fix $x_A = 0$, $x_B = 1$ and $p_j = p \in (c, V - t]$ for simplicity. The novel element is that each consumer $i \in [0, 1]$ needs to make a blocking decision $b_i \in \{0, 1\}$, simultaneous to the firms' choices of targeting strategies. Specifically, $b_i = 0$ means that the consumer does not block and decides as in the baseline model. If $b_i = 1$, any information received is blocked prior to inspection by the consumer, and i earns the reservation utility zero. Like Johnson (2013) we assume that each ad causes a nuisance to the consumer who receives it. Specifically, each consumer i bears a nuisance cost $\gamma_i \in [0, \bar{\gamma}]$ for each incoming message, where γ_i is privately known to i and an iid draw from a commonly known distribution $F(\cdot)$. Consumers thus face a trade-off between enduring ads and foregoing consumption utility, and we assume that consumers form rational expectations about their consumption utilities when making their blocking decision. Further, we impose the tie-breaking rule that whenever a consumer is indifferent, she will choose not to use any ad blockers. This conveniently allows us to ignore the trivial equilibria where almost all consumers block, and both firms target a zero-measure set of consumers.

Unlimited attention We first establish that without LA only non-intrusive targeting profiles can be sustained in equilibrium, and ad blockers will not be used.

Proposition 6 *Suppose that $R_i > 1$ $\forall i \in [0, 1]$ and $\bar{\gamma} \leq V - p - t/2$. Then any equilibrium of the targeting-blocking game features non-intrusive targeting strategies, i.e. $\lambda(\mathcal{I}_j \cap \mathcal{P}_{-j}) = 0$ and $\lambda(\mathcal{I}_j \cap \mathcal{P}_j) = \lambda(\mathcal{P}_j)$ $\forall j \in \{A, B\}$, and no consumer blocks information.*

The parametric assumption $\bar{\gamma} \leq V - p - t/2$ is with little loss of generality, and only rules out that some consumers always block unless they expect to receive no ad at all. As before, the fact that consumers have unbounded attention capacities disciplines firms from engaging into wasteful targeting. That is, with rational and attentive consumers the blocking threat becomes irrelevant because consumers will not be spammed in equilibrium, and hence blocking cannot be optimal. Proposition 6 resembles a result in Johnson (2013) who finds, in a different setting, that consumer blocking vanishes if the targeting abilities become sufficiently accurate.[42] We next establish that with LA equilibrium blocking arises despite *perfect* marketing data.

Limited attention We repeat the above analysis, the only change being that $R_i = 1 \; \forall i \in [0,1]$. We also assume, for the moment, that the perception probabilities are exogenous and equal for both firms ($\pi_A = \pi_B = 1/2$). The threat that consumers block as a consequence of mutual targeting diminishes the incentives for firms to choose an overlapping targeting strategy. Nevertheless, firms are locked in their roles as mass advertisers, despite substantial sale losses from blocking consumers, if the information cost is low enough.

Proposition 7 *Suppose that $R_i = 1 \; \forall i \in [0,1]$ and $c < \frac{pF(\gamma^*)}{2}$, where $\gamma^* = \frac{V-p-t/2}{2}$. In any equilibrium of the targeting-blocking game both firms behave as mass advertisers, i.e. $\lambda(\mathcal{I}_A) = \lambda(\mathcal{I}_B) = 1$, and a fraction $1 - F(\gamma^*)$ of consumers block.*

Consumers with low nuisance costs ($\gamma_i \leq \gamma^*$) do not mind receiving ads from both firms and therefore do not block. With low information costs and attention-constrained consumers, a firm can always secure a positive profit from targeting these consumers, independent of their locations and the competing firm's targeting strategy. Consequently, no firm would ever refrain itself from flooding the market with its ads, which makes annoyed consumers turn on the ad blockers.

[42] Equilibrium blocking in his model is a consequence of imperfect marketing data, and reflects the fact that some consumers receive pure spam as a consequence of targeting inaccuracy. We can easily replicate this finding in the current model (see section 3.2).

3.1 Ad blocking and salience competition

We now generalize the targeting-blocking model to the case of endogenous salience competition. Like in section 2.2.1, firms can choose how intensely to compete for attention. Thus, each firm j's strategy is again a pair of functions (g_j, s_j), but nuisance costs now increase in the exposure of consumers to attention-seeking activities. The probability that j is perceived by a jointly targeted, non-blocking consumer i is given by (6). Given firms' targeting-salience decisions, the nuisance experienced by a non-blocking consumer i is $\gamma_i S_i^\alpha$, where $S_i = \sum_j g_j(i)s_j(i)$ is i's exposure to attention-seeking and $\alpha \in (0,1)$ is a parameter. For tractability, we assume that the cost parameters γ_i are iid uniform on $[0,\bar{\gamma}]$. Simultaneous to the firms' choices, consumers decide on blocking based on the nuisance that they expect to experience if blockers are off. Firms face a strategic trade-off because the benefit of increased perception comes at an explicit salience cost and an implicit cost of increased blocking, where the strength of these effects depends on the equilibrium salience level. As before we suppose that generating a message at a salience level s costs a firm $h(s)$, where we impose the functional form $h(s) = \kappa s^\eta$, $\eta \geq 1$ and $\kappa > 0$, to obtain an explicit solution. Note that if $\alpha \to 0$ nuisance costs become insensitive to the salience levels, and we are essentially back to the setting of section 2.2.1.[43] Assuming that information provision is not too costly for any given salience level (i.e., κ is sufficiently small), we establish the following result.

Proposition 8 *There is $\hat{\kappa} > 0$ such that for any $\kappa \leq \hat{\kappa}$ an essentially unique equilibrium exists in which both firms behave as mass advertisers, i.e. $\lambda(\mathcal{I}_A) = \lambda(\mathcal{I}_B) = 1$, set the same salience*

[43]On the other hand, the information blocking model without salience competition can be viewed as the limiting case $\alpha \to 1$ of the current model, with the salience functions $s_A(i) = s_B(i) = 1$ $\forall i \in [0,1]$ being exogenously given.

level $s^{**} > 0$ for all consumers, and a fraction Λ of consumers use ad blocking tools, where

$$s^{**} = \left(\frac{(1-\alpha)p\mu}{2^{2+\alpha}\bar{\gamma}\kappa\eta} \right)^{\frac{1}{\eta+\alpha}}, \quad \Lambda = 1 - \frac{\mu}{\bar{\gamma}(2s^{**})^{\alpha}}, \quad \mu \equiv V - p - t/2. \tag{9}$$

The equilibrium has the following comparative statics:

Corollary 3 *Let $\kappa \leq \hat{\kappa}$. A decrease of κ and an increase of $\bar{\gamma}$ increases the fraction of blocking consumers and decreases firm profits.*

The intuition is as follows. A lower κ increases the intensity of salience competition because generating salience is less costly. Without the possibility of information blocking, it follows from Proposition 1 that this would not lead to a change in the firms' profit since the total equilibrium expenditures in advertising campaigns would remain unchanged. However, a more intense salience competition increases the exposure of consumers to advertising, resulting now in a smaller size of the market because more consumers block information. The strategic forces at work are such that this loss of sales cannot be compensated by the potential gains of cheaper information costs in equilibrium. Next, an increase of $\bar{\gamma}$ directly implies that the fraction of blocking consumers increases, because more consumers face higher nuisance cost, but it also has an indirect negative effect since firms save on costs and reduce their salience levels. Given that salience is decisive for attracting the attention of non-blocking consumers it is little surprising that the reduction in aggregate salience does not compensate for the increased propensity to block, leading to an increased equilibrium blocking fraction and lower profits.[44]

3.2 Privacy concerns

As a consequence of the above rational cost-benefit analysis, ad blockers are more likely to be switched on whenever consumers expect to receive a lot of only partly useful information. The mass advertising equilibrium triggered by consumer inattention is one reason for blocking, but

[44] Corollary 3 can be extended to the case of endogenous pricing as in section 2.2.2, where $p = V - t$ will be the (endogenous) price in the mass advertising equilibrium.

imperfect marketing data and the resulting equilibrium overlap (Proposition 4) may likewise induce blocking even with fully attentive consumers (see also Johnson, 2013). A number of recent advertising reports, mentioned in the introduction of this article, emphasize that consumers state privacy concerns among the main reasons why ad blockers are used, alongside with the nuisance of an increased exposure to ads. We now argue that privacy concerns jointly with a sentiment of information overload are more likely to be voiced if the information glut follows from strategic attempts to flood the market with information, rather than just reflecting poor firm-side marketing data.

To fix ideas, suppose that firms have imperfect marketing data as in section 2.2.3 and consumers are described by a nuisance cost of advertising as before. Additionally, consumers can communicate some private data, which are informative about their true tastes, to both firms at a cost $e > 0$ of giving up their privacy. For example, a consumer may allow firms to have access to her search history on Google, or generally accept (third-party) Cookies. In such a setting, the conventional prediction with attentive consumers is as follows. By Proposition 4, the equilibrium overlap of targeting strategies is reduced if consumers communicate their data to both firms. In particular, if firms can learn preferences from the private data, a consumer can make sure to receive only information from her prime firm by making her data publicly accessible. Accordingly, all consumers communicate their data to both firms if e is small enough, and neither privacy concerns nor ad blocking would arise in equilibrium. Further, no firm would invest into salience, and consumers receive just as much information as they need.

All of these results change with LA. With inattentive consumers and low information costs, mass advertising with equilibrium blocking remains the unique equilibrium prediction. As revealing own data does not help the consumer to find the best alternative, no rational consumer would ever wish to make her data available to firms for *any* $e > 0$. Hence, the model with LA predicts ad blocking to occur jointly with mass advertising and consumers that are exposed to salience competition who express their unwillingness to give up private information, consistent with the main reported consumer sentiments.

4 Conclusion

In terms of advertising expenditures, targeted advertising has been a key component in past years, but the real-world perception of how targeting has effectively worked out for firms and consumers is different from what theorists and practitioners have predicted. The strong expansion of the internet during the last two decades has allowed firms to obtain much preciser marketing data about consumers and facilitated targeting their adverts at certain consumer segments. Competitive forces then should lead firms to concentrate their targeting on the consumer segment with the highest willingness-to-pay for their products, leading to an informationally efficient segmentation of the market, where consumers obtain information predominantly about products which are most useful to them. Nevertheless, the advertising industry has witnessed consumers complaining about being overloaded with only partially useful ads and voicing privacy concerns, responding with an increasing usage of ad blocking software. Likewise, being on the consumers' mind remains a prime concern of the advertising firms. In this article we have shown that these key observations can be explained within a standard targeting framework once limited consumer attention is accounted for. The presence of enough inattentive consumers annihilates the existence of a segmenting equilibrium, because information dissemination to prime consumers ceases to work as a reliable shield against business stealing by the competitor if the consumer is not attentive to each incoming message. The ability to attract or retain attention further detaches the targeting choice from traditional fundamentals such as preferences or the degree of product differentiation, making firms target far less precisely than they could – in a highly inattentive world salience becomes king despite sophisticated targeting abilities and precise marketing data. It follows that the scope for targeting as an efficient and effective marketing instrument is much smaller, for both firms and consumers, once limited attention is taken into account. Given the many consequences caused by LA, future empirical work may want to take into account sensible measures to account for inattention when evaluating information and advertising policies.

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A Comments and further extensions

A.1 Hotelling framework

Our main results do not hinge on the specifics of the Hotelling framework. This can easily be seen as the proof of Theorem 1 does not use the line structure; it only requires that consumers are partitioned into prime segments. It follows that whenever preferences allow for a measurable assignment of prime segments to consumers and the set of indifferent consumers has zero measure, the equilibrium targeting strategies must show the paradigm stated in Theorem 1.

A.2 Pre-informed consumers

The qualitative results of Theorem 1 do not hinge on the assumption of ex ante uninformed consumers. Suppose that a subset $\hat{\mathcal{I}}_j \subset [0,1]$ of consumers has ex ante information about j. Hence if $i \in \hat{\mathcal{I}}_j$ but $i \notin \hat{\mathcal{I}}_{-j}$ firm j is i's default choice. With unbounded attention capacities, it follows from the proof of Theorem 1 that each firm will target the subset $\mathcal{P}_j \backslash \hat{\mathcal{I}}_j$ of consumers. Hence the resulting equilibrium has the partitional property known from Theorem 1 (i), but possibly at lower information expenditures, because no firm j would send a message to an attentive consumer $i \in \mathcal{P}_{-j}$ nor to an attentive consumer with $i \in \hat{\mathcal{I}}_j \cap \mathcal{P}_j$. This is different with inattentive consumers. For example, if $c_j < q\pi_j p_j$, $j = A, B$, then both firms set $\lambda(\mathcal{I}_j) = 1$ as in (ii), because the default assignment does not assure transaction, and intrusion becomes profitable like in the baseline model.

A.3 Obfuscation and targeting

Consider the baseline model of section 2.1 and suppose that the firms play a two-stage game, where consumers initially are fully attentive ($q = 0$), but in the first stage each firm may use some (possibly costly) obfuscation device with the effect that $q = 1$ results. Such obfuscation strategies could involve, e.g., increasing the mental load on consumers required to "decode" the product by making the product appear more complex (Ellison and Wolitzky, 2012; Hefti, 2015), by trapping consumer attention on intentionally designed webpages (Ellison and Ellison, 2009) or by trying to imitate the appearance of the competitor's products by using similar packaging etc. to persuade consumers that there is no need to pay close attention as existing alternatives are near substitutes. Let $V_A = V_B = V$, $x_A = 0$ and $x_B > 0$ as well as $\frac{c_B}{p_B} < \pi_B \leq 1 - \frac{x_B}{2}$ and $\pi_A \geq \frac{c_A}{p_A} + \frac{x_B}{2}$.[45] Then, it follows from Theorem 1 that the superior firm B would never wish to obfuscate the market while the inferior firm A always benefits from obfuscation. Provided that obfuscation is cheap enough, there is an (essentially) unique subgame-perfect equilibrium, where firm A obfuscates the market in the first stage, and both firms target the entire market in the second stage. A further observation worth mentioning is that in a *symmetric* environment, where $V_A = V_B$, $x_A = 1 - x_B$, $\pi_A = \pi_B$ and $p_A = p_B > 2c$ no firm has an incentive to obfuscate the market even if obfuscation were free.

A.4 Ad blocking and attention-gating platforms

The inefficiency that arises from consumer blocking identified in section 3 reflects that a scarce resource – attention – is over-utilized by the market. An information gate, such as a platform, could reduce the welfare loss attributed to blocking by diminishing the information exposure of consumers. To see this in the model of section 3, suppose that consumers have limited attention and all information is trafficked by a platform which restricts the displayed information to the messages of a single firm for each consumer. Because this reduces the effective exposure to ads, even attention-constrained consumers would be less inclined to use blocking tools. Indeed, the platform could work as a pricing device for the missing market for attention. For example, the platform could decide which firm to display as an outcome of an all-pay auction. With symmetric firms this would result in equal perception chances ($\pi_A = \pi_B = 1/2$) in the symmetric bidding equilibrium. In the initial model of section 3, such perception chances would induce more blocking in the mass advertising equilibrium compared to the platform, because all consumers would face a higher ad exposure without improved chances to perceive the right option. This simple role of the platform as an attention gate does not require that the platform holds any

[45] The last two inequality conditions hold if the information costs are sufficiently low and firm B is located sufficiently close to firm A.

additional information about consumers,[46] but blocking would be even further reduced if the platform could improve the chances of the idiosyncratically best products being shown to its users.

A.5 Location-depending salience effects

We modify the model with salience competition from section 2.2.1 by assuming that mutually targeted consumers with a stronger preference for a firm are more likely to pay attention to this firm for a given salience profile $(s_A(i), s_B(i))$.[47] Let

$$\Delta_i \equiv U_i(A) - U_i(B) = \begin{cases} t(x_A + x_B - 2i), & i \in [x_A, x_B] \\ t(x_B - x_A), & i < x_A \\ t(x_A - x_B), & i > x_B \end{cases}$$

denote the net utility difference. In the following, we suppose that how effective a firm's attention-seeking efforts are depends on the utility difference Δ_i, where the chance to attract consumer i's attention for a given salience profile increases (decreases) weakly in Δ_i for firm A (for firm B). For simplicity, we assume symmetric locations of firms ($x_A + x_B = 1$), remarking that the mass advertising result derived below does not hinge on this assumption. For any salience profile, perception chances of firm A are determined as

$$\pi_{Ai} = \begin{cases} \frac{s_A(i)y(\Delta_i)}{s_A(i)y(\Delta_i)+s_B(i)y(\Delta_i)^{-1}} & s_A(i) + s_B(i) > 0 \\ 1/2 & \text{otherwise,} \end{cases} \tag{A.1}$$

where $y : [t(2x_A - 1), t(1 - 2x_A)] \to \mathbb{R}_{++}$ is a non-decreasing function with $y(0) = 1$ and $y(-\Delta) = y(\Delta)^{-1}$. The function y thus captures how a pre-existing proclivity Δ in preferences affects the effectiveness of the attention-seeking salience stimuli. Perception chances of firm B are $\pi_{Bi} = 1 - \pi_{Ai}$. The symmetry embodied in y means that no firm has a systematic advantage

[46] This is different from the role of a platform as an information collector and broker that has been discussed in the literature (e.g., Bergemann and Bonatti, 2011; Eliaz and Spiegler, 2017).

[47] We are grateful to a referee of this journal who inspired the following extension.

in attention-seeking over the entire consumer population, while consumers closer to, say, firm A become more likely to pay attention to A ceteris paribus if, e.g., y is strictly increasing. Let $h(\cdot)$ be the a salience cost function as in section 2.2.1. To solve the model, let $y_i \equiv y(\Delta_i)$, and let $\sigma_A(i) \equiv s_A(i)y_i$ and $\sigma_B(i) \equiv s_B(i)/y_i$ denote effective salience efforts. Then, the firms compete in effective salience levels, where the payoffs from a mutually targeted consumer i are

$$\Pi_A(i) = \frac{\sigma_A(i)}{\sigma_A(i)+\sigma_B(i)}p - h\left(\frac{\sigma_A(i)}{y_i}\right) \qquad \Pi_B(i) = \frac{\sigma_B(i)}{\sigma_A(i)+\sigma_B(i)}p - h\left(\sigma_B(i)y_i\right), \tag{A.2}$$

and thus the optimal effective salience levels are implicitly determined by

$$\frac{\sigma_B(i)}{\left(\sigma_A(i)+\sigma_B(i)\right)^2}p = h'\left(\frac{\sigma_A(i)}{y_i}\right)\frac{1}{y_i} \qquad \frac{\sigma_A(i)}{\left(\sigma_A(i)+\sigma_B(i)\right)^2}p = h'\left(\sigma_B(i)y_i\right)y_i. \tag{A.3}$$

The following proposition establishes that firms behave as mass advertisers despite location-depending salience effects.

Proposition A1 *In any equilibrium of the targeting-salience game with location dependence, both firms behave as mass advertisers, i.e.,* $\lambda(\mathcal{I}_A) = \lambda(\mathcal{I}_B)$.

Proof: We show that both firms earn strictly positive profits from mutually targeted consumers if they optimize over their effective salience levels, hence mass advertising is the unique targeting equilibrium. Consider a mutually targeted consumer i. Then, the firms compete in effective salience levels $\sigma_A(i), \sigma_B(i)$ according to (A.2). Equilibrium existence of this type of game is known (see, e.g., Hefti, 2016). Further, firm A (likewise for B) earns a strictly positive profit by best-responding to $\sigma_B(i)$ iff

$$\frac{h'\left(s_A(i)\right)s_A(i)}{h\left(s_A(i)\right)} > \frac{\sigma_B(i)}{\sigma_A(i)+\sigma_B(i)}.$$

This inequality is satisfied by the convexity assumption imposed on $h(\cdot)$. ■

If salience costs are given by an arbitrary power function, $h(x) = x^\eta$, $\eta > 1$, more can be said about the salience intensities, the equilibrium perception probabilities and the equilibrium profits. While the effective salience levels σ_A, σ_B may differ for mutually targeted consumers as a consequence of the location-depending salience effects, both firms send the *same* number of attention-seeking messages s_A, s_B to such consumers, showing that the result of Proposition 1 about identical salience levels can also occur with location-dependence.[48]

Corollary A1 *If $h(x) = x^\eta$, then $\pi_A(y)$ and $\Pi_A(y)$ are strictly increasing in y, while $\pi_B(y)$ and $\Pi_B(y)$ are strictly decreasing in y in equilibrium. Further, $\sigma_A > \sigma_B$ iff $y > 1$, and $s_A(i) = s_B(i)$ for any mutually targeted consumer.*

Proof: Dividing the two optimality conditions (A.3) yields $\sigma_A(i) = y_i^2 \sigma_B(i)$. This directly implies that $s_A(i) = s_B(i)$ as well as that $\sigma_A(i) > \sigma_B(i)$ iff $y_i > 1$. Because

$$\pi_{Ai}(y) = \frac{1}{1 + \frac{\sigma_B(i)}{\sigma_A(i)}}$$

it also follows that $\pi'_A(y) > 0$ and $\pi'_B(y) < 0$. Likewise,

$$\Pi_A(i) = \pi_{Ai}(y) p \left(1 - \frac{1}{\eta}(1 - \pi_{Ai}(y))\right),$$

implying that $\Pi'_A(y) > 0$, and a similar calculation shows $\Pi'_B(y) < 0$. ■

[48] In campaign advertising of US elections it has been observed that campaign expenditures, while different for different states, show a strong symmetry between the republican and the democratic parties within states (see, e.g., Gallego and Schofield, 2017). The less decided a state appears to be, the higher the mutual advertising expenditures, whereas in decided states, both parties advertise at a lower intensity. If voters in the states are thought of being more or less aligned with a party in the spirit of the Hotelling line, the location-dependent salience model would predict such a pattern.

A.6 Persuasive advertising and limited attention

Previous research has sometimes considered a persuasive role of advertising (see Bagwell, 2007 for a survey).[49] This literature is marked by a substantial heterogeneity in what it effectively means to "persuade" (see Stigler and Becker, 1977 for an early critique of "persuasion"), ranging from mostly abstract shifts of the demand curve to very specific types of preferential shocks. It is conceivable that with utility-enhancing advertising a heavily advertised alternative is more likely to be chosen, which is the same type of prediction as obtained from our salience model of section 2.2.1, making both theories hard to disentangle on purely observational grounds.[50] The central question remains whether persuasive advertising and salience competition are strategically equivalent, meaning that they predict the same equilibrium outcome in the targeting context studied by this paper. We demonstrate, by means of a simple argument, why this most likely is not the case.

Suppose that ads targeted at consumers are capable of changing the individual valuations V_A, V_B. We concentrate on the simplest case where the ads of firm j increase V_j without further affecting V_{-j},[51] and abstract from price competition. We now argue that a model of "pure" persuasion with symmetric firms but otherwise fully attentive consumers would not result in intrusive targeting profiles because consumers are always capable of comparing all alternatives. Suppose that both firms are endowed with similar persuasion and perfect targeting abilities. Assume that, say, firm B targets a prime consumer i of firm A and manages to increase V_B to V_B' as a consequence of its persuasive efforts. Because $i \in \mathcal{P}_A$, it suffices for A to choose a persuasion intensity that yields the valuation $V_A' = V_B'$ to secure the demand of consumer i.[52] That is, firm A can effectively counter any intrusion by B with the same (or possibly even a lower) persuasion intensity. Because firm B thus has no reasonable hope to seize consumers

[49] We are grateful to a referee of this journal who inspired us to consider persuasive advertising within our targeting model.

[50] Field studies on persuasion cannot resolve this identification problem, but recent evidence from voting in US elections suggest that the effects from "persuasive" political ads are not long lasting (the effects wear off within a week) and exposure to ads matters (Gerber et al., 2011), both being explanations which seem more consistent with limited working memory and salience competition than with changes of the underlying preferences.

[51] This assumption is not critical as long as there is no binding outside option, as then only valuation *differences* $V_A - V_B$ matter for consumer choice.

[52] This follows because only valuation differences matter as long as the outside option does not bind. Also note that the outside option becomes less likely to bind given the assumption that advertising increases V_j.

from A's prime segment, firm B does not target that segment. It follows that mass advertising is an unlikely and the segmenting equilibrium is the likely outcome of such a game, in contrast to the limited attention prediction. In fact, Egli (2015) studies a related equilibrium model, where each of two firms can target a *single* location on the Hotelling line with a certain persuasion intensity. The targeted location biases the willingness-to-pay of close consumers towards the targeting firm at a rate which decreases in distance, similar to the waves emitted by a pebble thrown into a pond. In the equilibrium of his model, firms choose the *antipodal* locations as their targeted location, which leads to a segmented equilibrium, consistent with the intuition given above.

While a theory of targeting with purely persuasive advertising may fail to support the mass advertising equilibrium, persuasion and salience competition may complement each other. In particular, the ability to increase the individual willingness-to-pay became relevant in our model jointly with limited attention and price competition, because firms set monopoly prices in the mass advertising equilibrium which depend directly on the willingness-to-pay. Accordingly, firms profit if they manage to increase consumer reservation prices, and limited attention annihilates the comparison effect which destroys the mass advertising equilibrium in case of pure persuasion and unlimited attention capacities.

B Proofs

Proof of Theorem 1 Note that leaving a non-zero measure set of consumers in its prime segment uninformed about its product (i.e., $\lambda(\mathcal{I}_j \cap \mathcal{P}_j) < \lambda(\mathcal{P}_j)$) is never optimal for firm j if $c_j < q\pi_j p_j + (1-q)p_j$, since by covering these consumers firm j can always secure a positive expected payoff. But then, given that firms will fully cover their respective prime segments, there is no point for any firm j to penetrate its competitor's prime segment (i.e., $\lambda(\mathcal{I}_j \cap \mathcal{P}_{-j}) > 0$) if $c_j > q\pi_j p_j$, since the expected profit from doing so is negative. Thus, (i) immediately follows.

If $c_j < q\pi_j p_j$, then leaving any non-zero measure set of consumers uninformed about its product (i.e., $\lambda(\mathcal{I}_j) < 1$) would never be optimal for firm j, since by covering these consumers firm j can always secure a positive expected profit, from which (ii) follows.

Next, we prove (iii). Since $c_{-j} > q\pi_{-j}p_{-j} + (1-q)p_{-j}$, targeting any non-trivial set of consumers in $[0,1]$ will be profitable for firm $-j$ if and only if they are not targeted by firm j. If $c_j > q\pi_j p_j$ and $c_j < q\pi_j p_j + (1-q)p_j$, then as we have argued the latter inequality implies that it would never be optimal for firm j to leave a non-trivial set of consumers in $\mathcal{P}_j$ uninformed about its product. In addition, the condition $c_j > q\pi_j p_j$ implies that targeting any non-trivial set of consumers in $\mathcal{P}_{-j}$ will be profitable for firm j if and only if they are not targeted by firm $-j$. The first part of (iii) thus immediately follows. As for the second part, simply note that if we instead have $c_j > q\pi_j p_j + (1-q)p_j$, then the strategic consideration of firm j will be identical to that of firm $-j$: It is profitable to target a set of consumers in $[0,1]$ if and only if they are not targeted by its competitor.

Finally, for (iv), since as a dominant strategy firm j will target essentially the entire market if $c_j < q\pi_j p_j$, targeting consumers located in $\mathcal{P}_j$ will never be profitable for firm $-j$ given $c_{-j} > q\pi_{-j}p_{-j}$. If, in addition, $c_{-j} > q\pi_{-j}p_j + (1-q)p_{-j}$, then even targeting a consumer located in $\mathcal{P}_{-j}$ will not be profitable for firm $-j$. Hence, given firm j's dominant strategy, it is a best response for firm $-j$ to target only the consumers located in $\mathcal{P}_{-j}$ if $c_{-j} \in (q\pi_{-j}p_{-j}, q\pi_{-j}p_{-j} + (1-q)p_{-j})$, and it will optimally choose to shut down if $c_{-j} > q\pi_{-j}p_{-j} + (1-q)p_{-j}$. ■

Proof of Corollary 1 The statement follows from result (i) of Theorem 1. ■

Proof of Corollary 2 Replace "c_j" in the proof of Theorem 1 by "$c_j(q\pi_j + 1 - q)$". ■

Proof of Proposition 1 Given the arguments in the main text we only need to show that no asymmetric equilibria exists. First, consider a jointly targeted consumer $i \in \mathcal{I}_A \cap \mathcal{I}_B$. Then, there cannot be an equilibrium where $s_A^*(i) \neq s_B^*(i)$. This follows from a corollary (of Proposition 4) on symmetric contests in Hefti (2017), stating that best-reply maps associated with an equation like (7) can never possess asymmetric fix points. Because we take (7) to hold for any mutually targeted consumer it follows therefore that $s_A^*(i) = s_B^*(i) = s^*$ on any $\mathcal{I}_A \cap \mathcal{I}_B \neq \emptyset$ in equilibrium. ■

Proof of Proposition 2 Let $\tilde{\mathcal{P}}_A = [0, i_0]$ and $\tilde{\mathcal{P}}_B = [i_0, 1]$, where

$$i_0 = \max\left\{\min\left\{\frac{x_A + x_B}{2} + \frac{p_B - p_A}{2t}, 1\right\}, 0\right\}.$$

For a given pair of prices (p_A, p_B), it is straightforward to verify that the equilibrium targeting strategy of firm j in the targeting subgames must take on the following form. If $p_j < c$, then $g_j(i) = 0\ \forall i \in [0, 1]$; If $p_j = c$, then $g_j(i) \in \{0, 1\}$ if either $g_{-j}(i) = 0$ or $i \in \tilde{\mathcal{P}}_j$ and $g_j(i) = 0$ otherwise; If $p_j > c$, then $g_j(i) = 1$ if $p_j \leq V - |i - x_j|t$ and either $g_{-j}(i) = 0$ or $i \in \tilde{\mathcal{P}}_j$. Note that we need not worry about the tie-breaking rule for the marginal consumer i_0 because she has zero mass.

Lemma B1 In any SPE, $p_j > c\ \forall j \in \{A, B\}$.

Proof: Consider any (p_A, p_B) such that $p_A \leq c$, and hence $\Pi_A \leq 0$. If $p_B < c$, then $g_B(i) = 0$ $\forall i \in [0, 1]$, and A could secure a positive profit by deviating to $\tilde{p}_A = V - t$ and setting $g_A(i) = 1$ $\forall i \in [0, 1]$. If $p_B \geq c$, then by deviating to $\tilde{p}_A = c + \varepsilon$ for sufficiently small $\varepsilon > 0$, firm A can make a positive profit by targeting a fraction $i_0 > 0$ of consumers in the second stage. Hence, for (p_A, p_B) to be part of a SPE we must have $p_A > c$. The proof for $p_B > c$ is analogous. ■

Lemma B2 In any SPE, $t(x_A + x_B) - 2t < p_A - p_B < t(x_A + x_B)$.

Proof: If $p_A - p_B \geq t(x_A + x_B)$ then $i_0 = 0$ and $g_A(i) = 0\ \forall i \in [0, 1]$, since all consumers would find B the optimal choice and it is also optimal for firm B to set $g_B(i) = 1\ \forall i \in [0, 1]$ given $p_B > c$. Hence $\Pi(A) = 0$. But then, given $p_B > c$ in any SPE, firm A can profitably deviate to $p_A = c + \varepsilon < p_B$ for some $\varepsilon > 0$, because there then is a positive interval of consumers located around x_A that now find A to be the best choice. Hence, for (p_A, p_B) to be part of a SPE we must have $p_A - p_B < t(x_A + x_B)$. The proof for $p_A - p_B > 2t - t(x_A + x_B)$ is analogous. ■

Lemma B3 In any SPE, $p_A \leq V - t\max\{x_A, |i_0 - x_A|\}$, and $p_B \leq V - t\max\{(1 - x_B), |i_0 - x_B|\}$.

Proof: By Lemma B2, we have $i_0 \in (0, 1)$ in any SPE. Now suppose, in contradiction, that $p_A > V - t\max\{x_A, |i_0 - x_A|\}$ in some SPE. Then there must be a non-trivial interval of consumers in $[0, i_0]$ remain untargeted by firm A; otherwise the firm would incur a loss because all these consumers strictly prefer the outside option. However, because $i_0 \in (0,1)$ and all consumers with $i \leq i_0$ are A-captive once $g_A(i) = 1$, firm A is like a monopolist on $[0, i_0]$. But then, it follows from the assumption $V \geq c + 2t$ that firm A would always want to lower its price in the first stage and then serve all these consumers, regardless of firm B's corresponding targeting strategy. Therefore, we must have $p_A \leq V - t\max\{x_A, |i_0 - x_A|\}$ in any SPE. The proof for $p_B \leq V - t\max\{(1 - x_B), |i_0 - x_B|\}$ is analogous. ■

Given the characterization of the firms' targeting strategies in the second stage, the equilibrium prices can be found as a Nash equilibrium to the pure pricing game where each firm's demand is determined by the marginal consumer i_0 and payoffs are

$$\Pi_A = (p_A - c)\left(\frac{x_A + x_B}{2} + \frac{p_B - p_A}{2t}\right), \ \Pi_B = (p_B - c)\left(1 - \frac{x_A + x_B}{2} - \frac{p_B - p_A}{2t}\right), \quad \text{(A.4)}$$

subject to the restrictions on (p_A, p_B) imposed by the previous lemmata. It is straightforward to verify that this pricing game has a unique interior Nash equilibrium given by p_A^* and p_B^* as quoted by the proposition, which also satisfy lemmata B1-B3. We can thus conclude that (p_A^*, p_B^*) is the unique SPE price, and the market is segmented at the marginal consumer $i_0 = \frac{1}{3} + \frac{x_A + x_B}{6}$. The equilibrium profits can be obtained via plugging p_A^* and p_B^* into (A.4). ■

Proof of Proposition 3 Let $p_j^M = V - t\max\{x_j, 1 - x_j\}$ denote the hypothetical monopoly price each firm would set conditional on being a monopolist in the market facing a information cost of c. Denote $\mathcal{O}_j(p_j) = \{i \in [0,1] \big| V - p_j - t|i - x_j| \geq 0]\}$. By construction, any consumer $i \notin \mathcal{O}_j(p_j)$ will strictly prefer the outside option to the product offered by firm j.

(i) Suppose firm A sets $p_A = p_A^M$. Subsequently, it is a dominant strategy for firm A to target the entire market. Since in this case firm A's targeting decision will not be affected by firm B's price, it is clear that firm B's best response involves $p_B \in [p_B^M, V]$, which implies that the subsequent targeting decision of firm B is also pinned down by its choice of p_B. More specifically, firm B will optimally choose to target and only target the consumers located in $\mathcal{O}_B(p_B)$, and its expected payoff is given by $\Pi_B = \lambda(\mathcal{O}_B(p_B))(\pi_B p_B - c)$. Therefore, firm B's optimal choice of p_B requires it to solve a standard monopoly pricing problem, given its location x_B and the fact that any price it chooses will be discounted by π_B. It is then straightforward to verify that given $\pi_B(V - t(1+\max\{x_B, 1-x_B\})) > c$, the unique best response for firm B is to set p_B^M and also target the entire market subsequently. Applying the same argument to A given $p_B = p_B^M$ shows that indeed the strategy profile specified in (i) is part of a SPE. We now establish the uniqueness of the SPE outcome by a series of lemmas.

Lemma B4 In any SPE, $\pi_j p_j > c \; \forall j \in \{A, B\}$.

Proof: First, consider a pair of prices (p_j, p_{-j}) such that $\pi_j p_j \leq c$ and $\pi_{-j}p_{-j} > c$. With such prices, setting $g_{-j}(i) = 1 \; \forall i \in [0, 1]$ is a dominant strategy for firm $-j$ in the targeting stage, and then $\Pi_j \leq 0$. As a best response, firm j will shut down and earn a payoff of zero in the corresponding targeting subgames. But then deviating to the monopoly price p_j^M together with $g_j(i) = 1 \; \forall i \in [0, 1]$ is profitable for firm j, since by doing so it can earns at least an expected payoff of $\pi_j p_j^M - c > 0$. Hence, any price pair (p_j, p_{-j}) such that $\pi_j p_j \leq c$ and $\pi_{-j}p_{-j} > c$ cannot be part of a SPE.

Next, consider any prices (p_j, p_{-j}) such that $\pi_j p_j < c$ and $\pi_{-j}p_{-j} \leq c$. Since $\pi_j p_j < c$, any equilibrium in the corresponding targeting subgames must be non-overlapping. Thus, with such prices, firm $-j$ earns $\Pi_{-j} \leq p_{-j} - c$. But then by unilaterally deviating to p_{-j}^M in the pricing stage, firm $-j$ will optimally target the entire market in the targeting stage and force firm j to shut down. This is a profitable deviation for firm $-j$, since $p_{-j}^M - c > p_{-j} - c$ according to the parametric assumption in (i). As a result, any prices (p_j, p_{-j}) such that $\pi_j p_j < c$ and $\pi_{-j}p_{-j} \leq c$ cannot be part of a SPE.

Finally, the price pair (p_A, p_B) such that $\pi_j p_j = c \; \forall j \in \{A, B\}$ cannot be part of a SPE either. With such prices, given our tie-braking assumption there is a unique equilibrium in the targeting subgame, in which $g_j(i) = 1 \; \forall i \in [0, 1]$ and $j \in \{A, B\}$, i.e., both firms will target the entire market. In this case, the expected payoffs are zero for both firms. But then, an unilateral deviation of, say, firm A to setting $p_A = p_A^M$ together with $g_A(i) = 1$ $\forall i \in [0, 1]$ will yield at least an expected payoff of $\pi_A p_A^M - c > 0$ for firm A. ■

Lemma B5 In any SPE, $p_j \in [p_j^M, V]$ $\forall j \in \{A, B\}$.

Proof: By Lemma B4, we can restrict our attention the price pairs (p_A, p_B) such that $p_j > c/\pi_j$ for all $j \in \{A, B\}$. Note that conditional on $p_j > c/\pi_j$, firm j's targeting decision is completely pinned down by its own price p_j: in the targeting stage it is a dominant strategy for firm j to target (and only target) the consumers in $\mathcal{O}_j(p_j)$. But then, it is clear that any price $p_j \in (c/\pi_j, p_j^M)$ is dominated by p_j^M. ■

Lemma B6 In any SPE, all consumers in $[0, \min\{2x_A, 1\}]$ are targeted by firm A, and all consumers in $[\max\{0, 2x_B - 1\}, 1]$ are targeted by firm B.

Proof: By Lemmas B5 and B6, we can restrict attention to the cases where $p_j \in [p_j^M, V]$ $\forall j \in \{A, B\}$. Geometrically, this implies that in equilibrium, $\mathcal{O}_j(p_j)$ is an interval around x_j. Also note that there cannot be a non-zero measure set of consumers that are untargeted by any firm in equilibrium, since in this case some firm j will get an additional consumer for sure by marginally lowering its price and then targeting further in the second stage. This is profitable for firm j since $p_j^M \geq V - t \geq t + c > \lambda(\mathcal{I}_j(p_j))t + c$.

Now suppose, in contradiction, that there exists $\varepsilon > 0$ such that the consumers located in $[1 - \varepsilon, 1] \subseteq [x_B, 1]$ are targeted by firm A but not by firm B. Since $x_A < x_B$, this would imply that firm A are charging $p_A = p_A^M$ and targeting the entire market. But then, the unique best response of firm B would be to choose $p_B = p_B^M$ and target the entire market as well. Thus, in any SPE firm B must target all consumers in $[x_B, 1]$. By symmetry, consumers located in $[\max\{0, 2x_B - 1\}, x_B]$ will also get a positive payoff by transacting with firm B and, hence, they will be targeted by firm B as well. Proving that in any SPE firm A must target all consumers in $[0, \min\{2x_A, 1\}]$ is analogous. ■

Now suppose, without loss of generality, that in equilibrium firm A charges $p_A \in [p_A^M, V]$ and targets the consumers in $[0, \lambda_A]$, while firm B charges $p_B \in [p_B^M, V]$ and targets the consumers in $[1 - \lambda_B, 1]$, where $\lambda_A, \lambda_B \geq 0$. As argued, there cannot be a non-zero measure set of consumers that are untargeted by any firm in equilibrium, thus $\lambda_A + \lambda_B \geq 1$. Suppose further that both λ_A and λ_B are strictly less than one or, equivalently, that $p_j \in (p_j^M, V]$ $\forall j \in \{A, B\}$. By marginally lower its price and target further, firm j's marginal revenue is *at least* $\pi_j p_j^M$, while its marginal (opportunity) cost is given by $(\lambda_A + \lambda_B - 1)\pi_j t + (1 - \lambda_{-j})t + c$. Hence, a necessary condition to have $p_j \in (p_j^M, V]$ in equilibrium is

$$\pi_j p_j^M < (\lambda_A + \lambda_B - 1)\pi_j t + (1 - \lambda_{-j})t + c \; \forall j \in \{A, B\},$$

which further implies $\pi_A p_A^M + \pi_B p_B^M < t + 2c$. However, this inequality can never hold, since the parametric assumption in (i) asserts that $\pi_j p_j^M > \pi_j t + 2c$ $\forall j \in \{A, B\}$. ■

(ii) Suppose that for given prices (p_A, p_B) firm A targets the a set $\mathcal{I}_A \subset [0, 1]$ of consumers in the second stage. Because $\pi_B V < c$ it cannot be profitable for B to target any subset of consumers in $\mathcal{I}_A$ of positive measure. Therefore, there cannot be overlap in any SPE, and it immediately follows that any targeting profile that is part of a SPE must be essentially of the form that A targets a subset $\mathcal{I}_A \subset [0, 1]$ and B targets the subset $\mathcal{I}_A^C$. Knowing that it will be the monopolist in its respective segment, each firm then sets the monopoly price p_j^* corresponding to its anticipated targeting subset. Given that the subset $\mathcal{I}_A$ in the above discussion is arbitrary, we can also conclude that any non-overlapping targeting profile that partitions the unit interval together with the prices $p_j^* = \inf_{i \in \mathcal{I}_j}\{V - t|i - x_j|\}$ can be sustained as a SPE outcome. ■

(iii) Without loss of generality, let $j = A$ and $-j = B$. Since $c > \pi_B V$, B would be only interested in targeting i if $g_A(i) = 0$. Moreover, because $c < \pi_A(V - t\max\{x_A, 1 - x_A\})$ it is a dominant strategy for A to set $p_A = p_A^M$ and to target the entire market in the second stage. Hence, the unique SPE outcome is as stated in (iii). ■

Remark on tie-breaking rule We now show that result (i) of Proposition 3 remains valid for any arbitrary tie-braking rule given the additional assumption that $c \leq t$. With respect to the above proof of (i) we only need to show that $p_j = c/\pi_j$, $\forall j \in \{A, B\}$ cannot be part of a SPE. Suppose, in contradiction, that $p_j = c/\pi_j$, $\forall j \in \{A, B\}$ in some SPE. Note that by the parametric assumptions in (i) we have $c/\pi_j < V - 2t$, thus $p_j = V - 2t - \varepsilon_j$ for some unique $\varepsilon_j > 0$. Without imposing a specific tie-braking rule, any targeting profile (g_A, g_B) such that partitions the unit interval constitute a targeting equilibrium in the second stage. Now consider a deviation to $\hat{p}_j^M > c/\pi_j$. With such a price, $g_j(i) = 1 \ \forall i \in [0,1]$ becomes a dominant strategy in the subsequent targeting subgame. Hence, firm j can secure itself a deviation payoff of $\Pi_j^D \geq \pi_j p_j^M - c \geq \pi_j(V-t) - c$. Therefore, for $p_j = c/\pi_j$ to be part of a SPE it is necessary that such a deviation is not profitable. This requires that $\hat{\Pi}_j \geq \Pi_j^D$, where $\hat{\Pi}_j$ is the payoff received in the equilibrium of the targeting subgame following both firms choosing $p_j = c/\pi_j$. Since the equilibrium of the targeting subgame cannot be overlapping given $p_j = c/\pi_j \ \forall j \in \{A, B\}$, we have $\hat{\Pi}_A = \lambda\left(V - 2t - \varepsilon_A - c\right)$ and $\hat{\Pi}_B = (1-\lambda)\left(V - 2t - \varepsilon_B - c\right)$, where $\lambda \in [0,1]$ is the measure of consumers targeted by A. However, the inequalities $\hat{\Pi}_j \geq \Pi_j^D \ \forall j \in \{A, B\}$ together imply that $V - 2t - c - \lambda\varepsilon_A - (1-\lambda)\varepsilon_B \geq V - t - 2c$ or equivalently that $c \geq t + \lambda\varepsilon_A + (1-\lambda)\varepsilon_B$, contradicting the assumption that $c \leq t$. ■

Proof of Proposition 4 Conditional on $z_{ji} = 1$, by Bayes' rule the probability that $j \succ_i -j$ is given by $1/(1+\alpha_j) \geq 1/2$. Since $p_j > (1+\alpha_j)c_j$, firm j's expected profit from targeting such a consumer is at least $p_j/(1+\alpha_j) - c_j > 0$. Hence, in any targeting equilibrium, $z_{ji} = 1 \implies g_j(i) = 1$ for almost every $i \in [0,1]$. Now suppose $z_{ji} = 0$. In this case, firm j knows for sure that $-j \succ_i j$. But then, firm j also knows that $z_{-ji} = 1$ with probability one and, hence, this consumer will be targeted by firm $-j$ almost surely. Given that consumers' attention is unconstrained, firm j can never gain anything by targeting such consumers. As a result, we have that in any targeting equilibrium, $z_{ji} = 0 \implies g_j(i) = 0$ for almost every $i \in [0,1]$. ■

Proof of Proposition 5 Identical to the proof of case (ii) of Theorem 1. ■

Proof of Proposition 6 Clearly, in any equilibrium $\lambda(\mathcal{I}_j \cap \mathcal{I}_{-j} \cap \mathcal{P}_j) = 0 \ \forall j \in \{A, B\}$, since targeting is costly and firm $-j$ can never win any consumer that is located in $\mathcal{P}_j$ and is also targeted by firm j. This implies that the targeting profile must be non-overlapping in any equilibrium, and therefore each consumer will be targeted by at most one ad. Next, suppose a consumer $i \in \mathcal{P}_j$ is targeted by firm $-j$. This would be profitable for firm $-j$ if and only if this consumer is not targeted by firm j and the probability that she will use the ad blocker is sufficiently low. But then, it would also be profitable for firm j to target this consumer, since in that case it will be the chosen firm if that consumer indeed dose not use the ad blocker. Hence, in equilibrium there cannot be a non-zero measure set of consumers in $\mathcal{P}_j$ being targeted by firm $-j$ only. Thus, in any equilibrium the targeting profile must be non-intrusive, and any consumer with rational expectation would not choose to use the ad blocker given the assumptions that $x_A = 0$, $x_B = 1$ and $\bar{\gamma} \leq V - p - t/2$, because in this case

$$EU_i(b_i = 0|\gamma_i) = V - p - \min\{i, 1-i\}t - \gamma_i \geq V - p - t/2 - \bar{\gamma} \geq 0.$$

This in turns implies that in any equilibrium, each firm j will target and only target consumers located in $\mathcal{P}_j$ (up to zero-measure sets). ■

Proof of Proposition 7 First, note that if a consumer with $\gamma_i \leq \gamma^*$ would not use the ad blocker even if she expects to be spammed by both firms, since:

$$EU_i(b_i = 0|\gamma_i) = V - p - \left(\frac{i}{2} - \frac{1-i}{2}\right)t - 2\gamma_i \geq V - p - \frac{t}{2} - 2\gamma^* = 0.$$

As a result, any firm can secure an expected revenue of $\frac{pF(\gamma^*)}{2}$ by targeting an arbitrary consumer, independent of the competing firm's targeting strategy. Hence if $c < \frac{pF(\gamma^*)}{2}$ holds, leaving a non-zero measure set of consumers untargeted would never be optimal for any firm, and we thus have $\lambda(\mathcal{I}_A) = \lambda(\mathcal{I}_B) = 1$ in any equilibrium. Consequently, in equilibrium every consumer rationally expects to receive ads from both firm and pay a transportation cost of $t/2$ if she does not switch on the ad blocker. It then follows that in any equilibrium, almost every consumer with $\gamma_i > \gamma^*$ will choose to use the ad blocker, while the remaining ones will not. ■

Proof of Proposition 8 First, there cannot be an equilibrium in which a non-zero set of consumers remains completely uninformed because consumers located in this set will not block and therefore any firm could capture these consumers by sending them a message at zero salience costs. Second, there cannot be any equilibrium in which a non-zero set of consumers is targeted by one firm only. This is because in such an equilibrium optimality requires the salience level of the messages received by such consumers to be zero, giving the competitor an incentive to steal away these consumers by sending them a message with an arbitrarily low salience level. It follows that the only type of equilibrium candidate is where $\lambda(\mathcal{I}_A) = \lambda(\mathcal{I}_B) = 1$. Consider a consumer $i \in [0, 1]$ who is aware of her LA and expects to receive messages from both firms at an aggregate salience level $S > 0$. This consumer's expected consumption utility, conditional on not blocking, is $V - p - t/2$. Thus, with rational expectations, the chance that i chooses to block is

$$\Pr(b_i = 1|S) = 1 - \Pr\left(\gamma_i \leq \frac{V - p - \frac{t}{2}}{S^\alpha}\right) = \begin{cases} 1 - \frac{\mu}{\bar{\gamma}S^\alpha} & S^\alpha > \frac{\mu}{\bar{\gamma}}, \\ 0 & \text{otherwise.} \end{cases}$$

With salience levels $s_A(i), s_B(i)$ such that $S^\alpha > \frac{\mu}{\bar{\gamma}}$, firm j's expected profit from i is

$$\Pi_j(i) = \frac{p\mu}{\bar{\gamma}(s_j(i) + s_{-j}(i))^\alpha} \cdot \frac{s_j(i)}{s_j(i) + s_{-j}(i)} - \kappa s_j(i)^\eta, \tag{A.5}$$

with first-order conditions given by

$$\frac{p\mu\,(s_{-j}(i) - \alpha s_j(i))}{\bar{\gamma}(s_j(i) + s_{-j}(i))^{2+\alpha}} = \kappa s_j(i)^{\eta-1}\eta. \tag{A.6}$$

The single symmetric solution to the first-order conditions of the two firms is $s_A(i) = s_B(i) = s^{**}$, where s^{**} is given by (9). To show that both firms sending their messages at the uniform salience level s^{**} to the entire market is an equilibrium it remains to verify that indeed $\mu/(\bar{\gamma}(2s^{**})^\alpha) \leq 1$ and firms make a positive expected profit from each mutually targeted consumer. Using (9) the first inequality becomes

$$\left(\frac{(1-\alpha)p}{2^{1+\alpha}\kappa\eta}\right)^\alpha \geq \left(\frac{\mu}{\bar{\gamma}}\right)^\eta \quad \Leftrightarrow \quad \kappa \leq \hat{\kappa} \equiv \left(\frac{\mu}{\bar{\gamma}}\right)^{\alpha/\eta} \frac{(1-\alpha)p}{2^{1+\alpha}\eta}.$$

Thus, given that $\kappa \leq \hat{\kappa}$ and both firms target consumer i with the salience level s^{**}, the expected profit of j is

$$\Pi_j(i) = \frac{p}{2}\frac{\mu}{\bar{\gamma}(2s^{**})^{\alpha}} - \kappa(s^{**})^{\eta}.$$

It is straightforward to verify that $\Pi_j(i) > 0 \Leftrightarrow 2\eta > 1 - \alpha$, where the last inequality holds since $\eta \geq 1$ and $\alpha > 0$. Therefore, it is indeed an equilibrium for both firms to target the entire market at s^{**}, provided that $\kappa \leq \hat{\kappa}$, and a fraction $\Lambda = 1 - \omega/(\bar{\gamma}(2s^{**})^{\alpha})$ of consumers (i.e., the ones with low nuisance tolerance) uses the blocking tools.

For uniqueness, it suffices to show that the best-reply function $s_j(s_{-j})$, implicitly defined by (A.6), and its counterpart $s_{-j}(s_j)$ do not have any asymmetric fixed points.[53] To prove this we make use of Theorem 2 in Hefti (2017), stating that if $s_j'(s_{-j}) > -1 \; \forall s_{-j} > 0$ then there cannot be such asymmetric fixed points. Let $j = A$ and $-j = B$. Note first that for any given $s_B > 0$, a solution $s_A(s_B)$ to the equation (A.6) always exists. Moreover, this solution must satisfy $0 < \alpha s_A(s_B) < s_B$ and must be unique because the LHS of (A.6) is decreasing in s_A whenever $s_B \geq \alpha s_A$. Now let

$$\phi(s_A, s_B) \equiv p\frac{\mu\,(s_B - s_A\alpha)}{\bar{\gamma}(s_A + s_B)^{2+\alpha}} - \kappa {s_A}^{\eta-1}\eta.$$

It follows that $\phi'_{s_A}(s_A(s_B), s_B) < 0$.[54] An application of the Implicit Function Theorem then yields that for given $s_B > 0$,

$${s_A}'(s_B) = -\frac{\phi'_{s_B}(s_A(s_B), s_B)}{\phi'_{s_A}(s_A(s_B), s_B)} = \frac{(1+\alpha)s_A\,(s_A(1+\alpha) - s_B)}{(\eta-1)(s_A+s_B)(s_B - \alpha s_A) - (1+\alpha)s_A(\alpha s_A - 2s_B)},$$

where the denominator is strictly positive (a consequence of $\phi'_{s_A}(s_A(s_B), s_B) < 0$). The condition $s_A'(s_B) > -1$ then can be reduced to $(1+\alpha)s_A > (\eta-1)(\alpha s_A - s_B)$, which is satisfied as $s_A > 0$

[53]Note that the j's best-reply function $s_j(s_{-j})$ is defined by (A.6) only if $(s_j(s_{-j}) + s_{-j})^{\alpha} \geq \mu/\bar{\gamma}$. If this inequality is violated, consumer i does not block for sure and the profit function is $\Pi_j(i) = p\frac{s_j(i)}{s_j(i)+s_{-j}(i)} - \kappa s_j(i)^{\eta}$, with associated first-order condition of the form (7). We already know from the proof of Proposition 1 that the corresponding best-reply map cannot have any asymmetric fixed point, and the fact that there is possibly an isolated non-differentiability of $s_j(i)(s_{-j}(i))$ at the switching point is irrelevant in order to exclude asymmetric fixed points by the result in Hefti (2017).

[54]Meaning: The partial derivative of $\phi(s_A, s_B)$ with respect to s_A, evaluated at $s_A = s_A(s_B)$. Note that $\phi'_{s_A}(s_A(s_B), s_B) < 0$ also assures that the underlying objective function is strongly quasiconcave in s_A.

and $s_B > \alpha s_A$. ■

Proof of Corollary 3 Using $s_A = s_B = s^{**}$ from (9) in (A.5) and rearranging gives

$$\Pi = \kappa^{\frac{\alpha}{\eta+\alpha}} \left(\frac{2\eta}{1-\alpha} - 1 \right) \left(\frac{(1-\alpha)p\mu}{2^{2+\alpha}\bar{\gamma}\eta} \right)^{\frac{\eta}{\eta+\alpha}}.$$

Hence $\frac{\partial \Pi}{\partial \bar{\gamma}} < 0$ because $\frac{2\eta}{1-\alpha} - 1 > 0$, and $\frac{\partial \Pi}{\partial \kappa} > 0$ is obvious. Further, we have

$$\Lambda = 1 - \frac{1}{2^\alpha} \left(\frac{2^{2+\alpha}\kappa\eta}{p(1-\alpha)} \right)^{\frac{\alpha}{\eta+\alpha}} \left(\frac{\mu}{\bar{\gamma}} \right)^{\frac{\eta}{\eta+\alpha}}$$

from which $\frac{\partial \Lambda}{\partial \bar{\gamma}} > 0$ and $\frac{\partial \Lambda}{\partial \kappa} < 0$ follow. ■